def^n impt.

general

examples

titles

solution methods

To solve DE's, have seen:

1. Find n indpt. sol^n's & combine
2. Found sol^n's using an n-term recurrance relation

For both, linearity is impt.

Will look @ sol^n's of eq^n's of inhomogenous form: $\mathcal{L}y(x) = f(x)$

$f(x)$ is prescribed f^n

$y(x)$ satisfying boundary conditions

kinda impossible to find particulars of

use linearity of $\mathcal{L}$ by building up required sol^n's as superposition of some set of f^n's each individually satisfying BC.

$\mathcal{L}\underbrace{y_i(x)}_{\text{eigenfunction}} = \lambda_i y_i(x)$, similar to eigenvectors $\vec{x}^i \to A\vec{x}^i = \lambda\vec{x}^i$

17.1 Sets of Functions

consider infinite-dimensional space of all reasonably well behaved f^n's $\underbrace{f(x), g(x), h(x), \ldots}$ on interval $[a, b]$

these f^n's form a LVS

set closed if: ① additivity ② multiplication ③ null vector ④ unity ⑤ negative f^n

now introduce set of linearly indpt. basis f^n's $y_n(x)$ $(n = 1, 2, \cdots, \infty)$ s.t. any reasonable f^n's in $[a, b]$ can be expressed

$$f(x) = \sum_{n=0}^{\infty} c_n y_n(x)$$

inner product w/ weight f^n $\rho(x)$

$$\langle f | g \rangle = \int_a^b f^*(x)\, g(x)\, \rho(x)\, dx \to \overset{\text{if orthogonal}}{= 0}$$

norm:

$$||f|| = \langle f | f \rangle^{1/2} = \left[\int_a^b f^*(x) f(x) \rho(x)\, dx \right]^{1/2} = \left[\int_a^b |f(x)|^2 \rho(x)\, dx \right]^{1/2}$$

called a Hilbert space

choose basis of linearly indpt. fn's, orthogonal, $\hat{\phi}_n(x)$ $(n = 1, 2, \ldots)$

$$\langle \hat{\phi}_i | \hat{\phi}_j \rangle = \int_a^b \hat{\phi}_i^*(x) \hat{\phi}_j(x) \rho(x)\, dx = \delta_{ij}$$

if $y_n(x)$ is linearly indpt. but not orthonormal, basis for Hilbert space, can make orthonormal set of basis fn's $\hat{\phi}_n$

$$\phi_0 = y_0$$

$$\phi_1 = y_1 - \hat{\phi}_0 \langle \hat{\phi}_0 | y_1 \rangle$$

$$\vdots$$

$$\phi_n = y_n - \hat{\phi}_{n-1} \langle \hat{\phi}_{n-1} | y_n \rangle - \cdots - \hat{\phi}_0 \langle \hat{\phi}_0 | y_n \rangle$$

Hilbert - Shmidt

find ϕ_n, normalize $\rightarrow \hat{\phi}_n$

11.2 Adjoint, Self-Adjoint, Hermitian Operator

adjoint: $\mathcal{L}^+$

$$\int_a^b f^*(x) [\mathcal{L}\, g(x)]\, dx = \int_a^b [\mathcal{L}^+ f(x)]^* g(x)\, dx + \text{boundary terms}$$

$\mathcal{L}$ $\rightarrow$ linear differential operator

$\mathcal{L}^+$ $\rightarrow$ its adjoint

$\rightarrow$ evaluate w/ integration by parts

self-adjoint if: $\mathcal{L} = \mathcal{L}^+$

if w/ $f(x)$ & $g(x)$ help satisfy BC

$$\rightarrow \int_a^b f^*(x) [\mathcal{L}\, g(x)]\, dx = \int_a^b [\mathcal{L}^+ f(x)]^* g(x)\, dx$$