



# Lab 6: Pulsed Nuclear Magnetic Resonance

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## 1 Background

A nucleus in an external magnetic field can be approximated as a spinning magnetic bar. For the behavior of a bar magnet with a dipole moment  $\vec{\mu}$ , the torque due to an external magnetic field  $B$  is  $\vec{\tau} = \vec{\mu} \times \vec{B}$ . When  $\vec{\mu}$  and  $\vec{B}$  are aligned, the energy is minimized as  $E_{mag} = \mu B$ . Adding some angular momentum to the bar magnet,  $\vec{L}$ , will have the bar magnet be unaligned with the magnetic field, causing some precession along the axis of  $B$ . The angular frequency of this precession is given by Equation 1

$$\vec{\omega} = \frac{\mu}{L} \vec{B} = \gamma \vec{B} \quad (1)$$

in which we define the gyromagnetic ratio as  $\gamma = \mu/L$ .

Similarly for a proton in an external magnetic field, some quantum angular momentum  $\vec{I}$ , due to its intrinsic nuclear spin, will cause the proton to rotate around an external magnetic field  $\vec{B}$ . Here, we can describe  $\vec{B} = B_0 \hat{z}$ , and  $I$ , instead of being described by some continuous property, is described in quantizations,  $I_z = m_l \hbar$ . Here  $m_l$  is the magnetic quantum number spanning values between  $-s$  and  $s$  in integer steps, where  $s$  is the intrinsic spin. Specifically for protons, we have  $s = 1/2$ , so  $m_l = \pm 1/2$ . We can write the energy for the proton's spin states similarly to that of the magnetic bar, as described in equation 2

$$E_{mag} = -\gamma \vec{I} \cdot \vec{B} = -\gamma I_z B_0 = -\gamma m_l \hbar B_0 \quad (2)$$

The energy separation from the two  $m_l$  energy states is given by equation 3

$$\Delta E_{mag} = (-\gamma(-1/2)\hbar B_0) - (-\gamma(1/2)\hbar B_0) = \gamma \hbar B_0 = \hbar \omega \quad (3)$$

We can tune the magnetic field to cause the proton to precess at some angular frequency  $\omega$ . We can then send pulses of radio frequency at that same frequency to have pulsed nuclear magnetic resonance. Using specific pulses, we can measure different characteristic times about how a material relaxes back to its equilibrium position with respect to the axis in which the magnetic field is precessing and the plane spanning the perpendicular direction from that axis.

## 2 Setting Resonance

To observe pulsed nuclear magnetic resonance, we send pulses of radio frequency that are in resonance with the precessing angular frequency due to the external magnetic field. After setting the frequency in the RF to around 15 MHz, we tune the current of the electro-magnet power supply to change the electro-magnet's magnetic field such that we can start seeing some of the beats produced by mixing both RF frequency and the material's processing frequency. For more precise tuning to resonant frequency, we would change the RF frequency. For resonance, we would expect this resonance to produce a signal near zero with little to no beats.

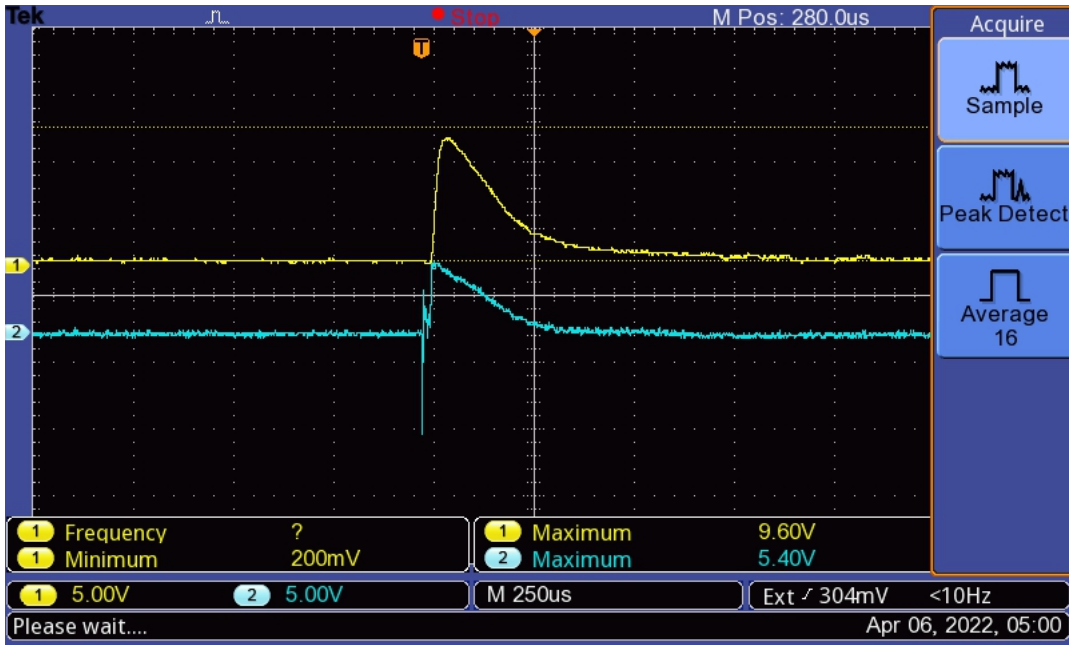


Figure 1: Resonance

Figure 1 is an example of reaching resonance, where the beats in the blue channel are drowned out. This signal is the free induction decay.

### 3 Gyromagnetic Ratio

To calculate the gyromagnetic ratio of glycerin and teflon nuclei, we calculate the magnetic field at which the material is in resonance with the RF pulses sent through, and use equation 4 to find the ratio, which is just a re-writing of 3.

$$\omega = \gamma B_0 \quad (4)$$

Isolating the gyromagnetic ratio, it is just the ratio of frequency to the magnetic field that the material is in.

Before making calculations, we first calibrated our Magnetometer with which we will be measuring the magnetic field. We used the calibration set from the lab to measure the magnetic flux that is measured by the device.

To make a calibration, I completed a linear fit to the following equation.

$$B_{meas} = mB_{real} + b \quad (5)$$

Where the  $B_{meas}$  is the magnetic field that I would measure with the magnetometer and  $B_{real}$  is the actual magnetic field from the calibration set. Using 5, I could make a fit using the measured magnetic field and its uncertainty to what should be set. We completed 5 different measurements for each of the 3 calibration devices. The uncertainty in the  $B_{meas}$  was the values that it would hover around as we allowed the measurement to be set, which would range between 1 and 3 Gauss for different measurements.

Manipulating the equation, we find the actual magnetic field, given our measured magnetic field, and its associated uncertainty in equations 6 and 7

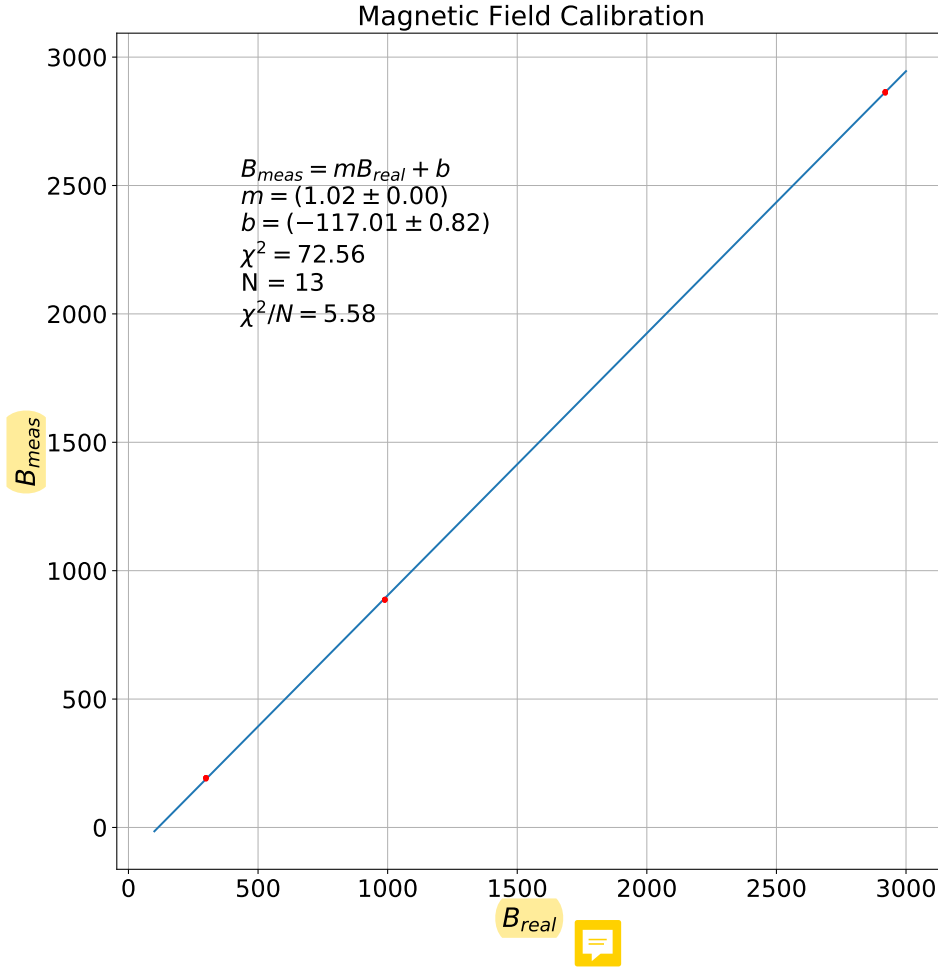


Figure 2: Calibrating B

$$B_{real} = \frac{B_{meas} - b}{m} \quad (6)$$

$$\Delta B_{real} = B_{real} \sqrt{\left(\frac{\Delta B_{meas}}{B_{meas} - b}\right)^2 + \left(\frac{\Delta b}{B_{meas} - b}\right)^2 + \left(\frac{\Delta m}{m}\right)^2} \quad (7)$$

We can find  $\Delta b$  and  $\Delta m$  by completing fits to our data. Using these equations to get the actual magnetic field, we can now turn to finally calculating the gyromagnetic ratio for each of the materials. To calculate it and its associated uncertainty, we use the following two equations.

$$\gamma = \omega_0/B_0 \quad (8)$$

$$\Delta\gamma = \gamma \sqrt{\left(\frac{\Delta\omega_0}{\omega_0}\right)^2 + \left(\frac{\Delta B_0}{B_0}\right)^2} \quad (9)$$

We completed three separate magnetic field measurements when at resonance. To find one final value of the gyromagnetic ratio for each of the materials, we average the three ratios, which is described (as well as its uncertainty) in equations 10 and 11.

$$\gamma_F = (\gamma_1 + \gamma_2 + \gamma_3)/3 \quad (10)$$

$$\Delta\gamma_F = \frac{1}{3}\sqrt{(\Delta\gamma_1)^2 + (\Delta\gamma_2)^2 + (\Delta\gamma_3)^2} \quad (11)$$

The measurement of the frequency goes out to a thousandth of a Megahertz, so we use an uncertainty of 0.001 MHz. For glycerin, we established resonance at 15.407 MHz. For teflon, resonance was established at 15.430 MHz. The following table summarizes the two calculated ratios

Material/Nucleus	Calculated $\gamma$ (MHz/Tesla)	Literature $\gamma$ (MHz/Tesla)
Glycerin / Hydrogen	$42.55 \pm 0.06$	$42.50 \pm 0.03$
Teflon / Flouride	$40.25 \pm 0.07$	$40.05 \pm 0.04$

Table 1: Gyromagnetic Ratio

For glycerin, our calculated gyromagnetic ratio is within uncertainty of the literature value. For teflon, the gyromagnetic ratio is slightly over the literature value and outside of the literature's uncertainty. Despite being outside of uncertainty, the value is very close.

## 4 Calculating Characteristic Times

To measure the characteristic time  $T_1$ , we first established resonance at the specific delay time at which our pulses are leaving our pulse programmer. We would then measure the height of the peak of the free induced current, seen in the following figure in yellow. We would measure its displacement from the value at which it asymptotes off to. In figure 4, the curve peaks at 7.00 Volts and asymptotes off to 1.00 Volts, so our change in voltage is 6.00 Volts. This change in voltage is what we would measure as a proxy for the net magnetization for the material.

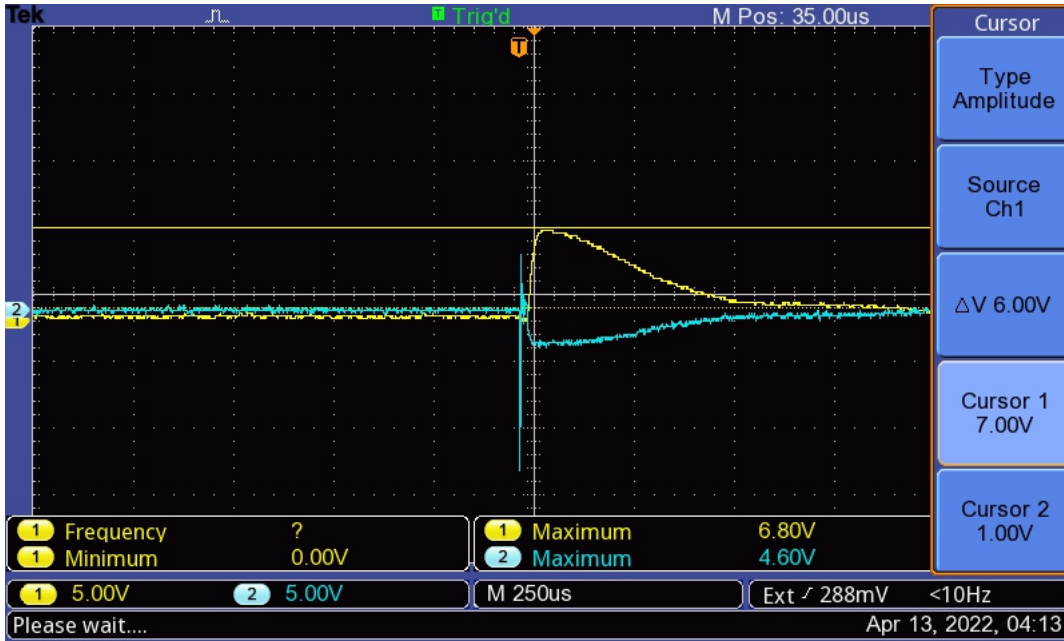


Figure 3:  $M_z$  Measurement

Using this net magnetization  $M_z$ , we can measure this again, but for a different delay time  $\tau$ . We can plot out the associated free induction decay peak/amplitude against the delay time, and then fit to an exponential curve described by the following equation.

$$M_z(\tau) = M_0(1 - 2e^{-\tau/T_1}) + C \quad (12)$$

The constant at the end of the equation is to account for the fact that applying the first pulse will not necessarily drop out the FID to zero, there is a bit of spill over. Applying this constant term accounts for this spill over.

Since we were only measuring the positive values, when plotting, we had to find a turnaround delay time at which the magnetization turns from negative to positive. We estimated this delay time at around 17 ms. Plotting out the values, we were able to confirm the general fit to the curve. We estimated the uncertainty of the measurement of the magnetization to be 0.2 Volts, as this was the lowest increment at which we could measure a change in voltage. Even when zooming in for a more precise measurement into the tenths of a Voltage, the fluctuations of the measurement were still in the order of 0.2 Volts.

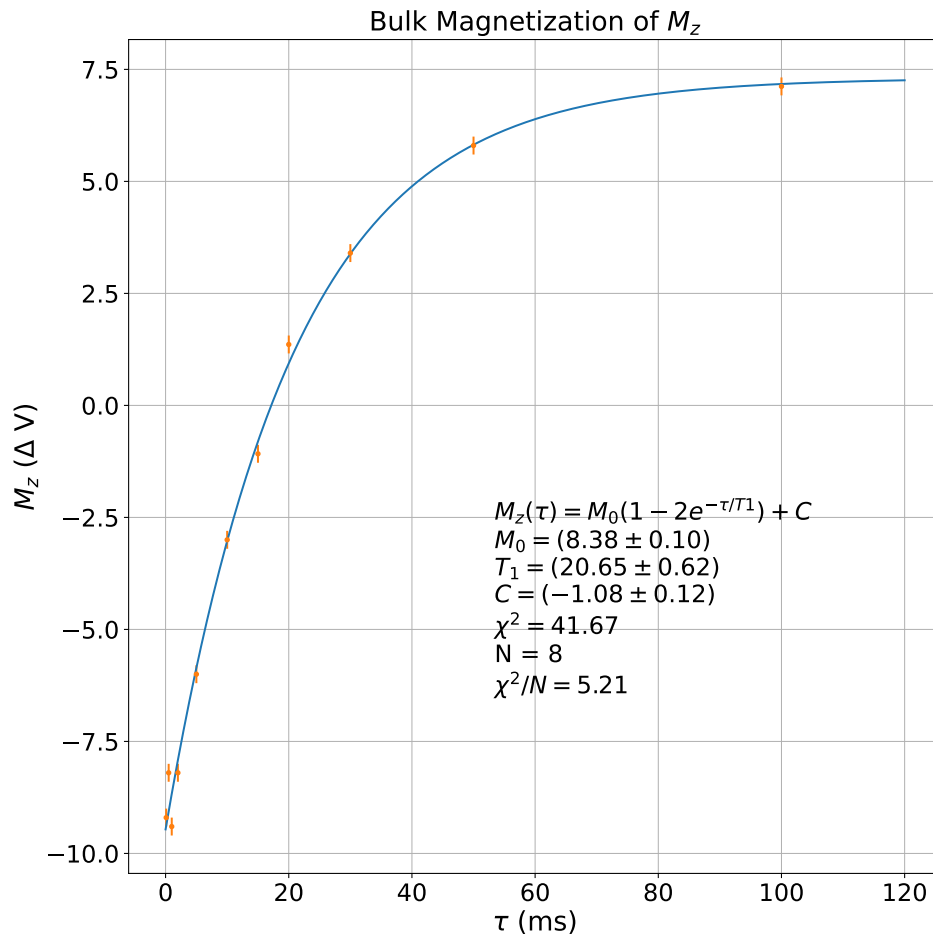


Figure 4: Bulk Magnetization of  $M_z$

We can complete a similar procedure, but now with different pulses. Initially to measurements of  $M_z$  had pulses that were first an  $\pi/2$  then a  $\pi/4$  rotation from the pulse, this would measure

the relaxation time of the spin-lattice relaxation time. Now  $T_2$  measure the spin-spin characteristic relaxation time  $T_2$ , we want to see how long it takes for the x-y plane (plane perpendicular to magnetic field) to relax. In this case, we will send through a  $\pi/4$  pulse, then a  $\pi/2$  pulse.

Now we can fit to equation 13 to try to find the characteristic time  $T_2$ . Likewise for the previous equation, the constant takes into account any spill over

$$M_{xy}(2\tau) = M_0 e^{-2\tau/T_2} + C \quad (13)$$

Again, we estimate the uncertainty in  $M_{xy}$  measurements as 0.2 Volts as that was the lowest increment for most of the measurements made, as well as the fluctuations when we did zoom in a bit. The fitted values are shown in the following figure.

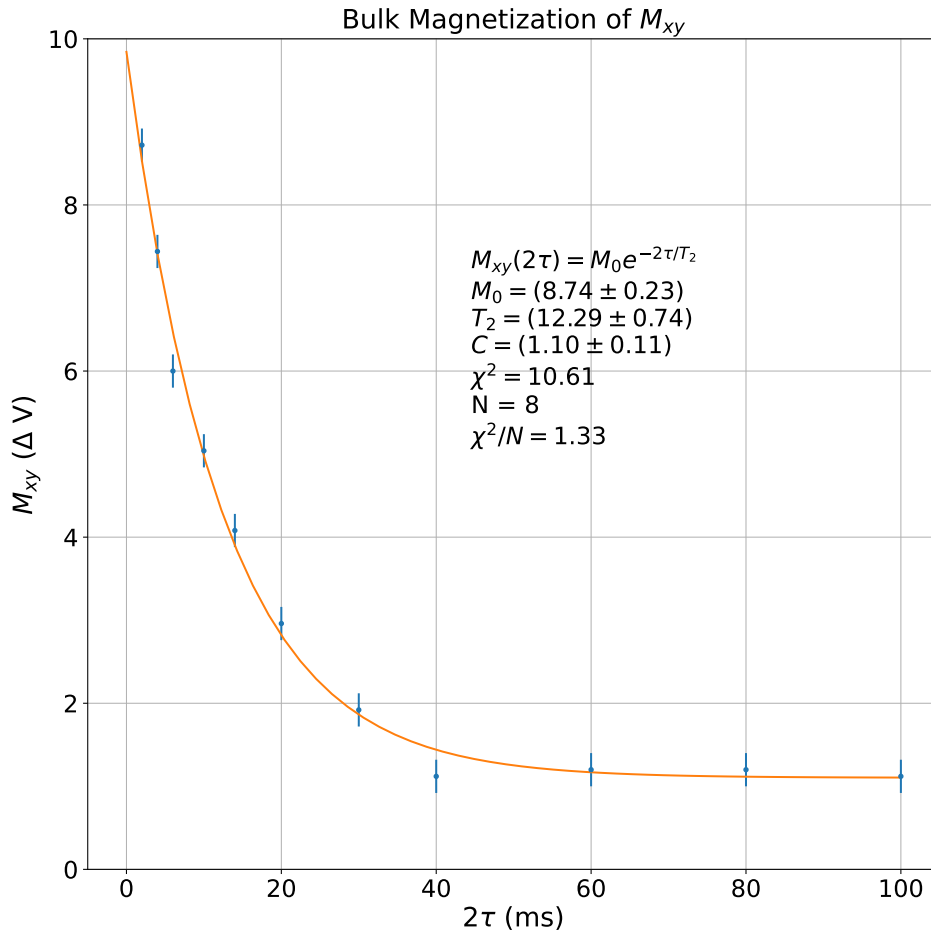


Figure 5: Bulk Magnetization of  $M_{xy}$

Characteristic Time	$\tau$ (ms)	$\chi^2/N$
$T_1$	$20.65 \pm 0.62$	5.21
$T_2$	$12.29 \pm 0.74$	1.33

Table 2: Characteristic Times

The reduced chi-squared for  $T_1$  is quite large despite being generally good fits for the data. This

may be due to the many points we took near a delay time of 0 ms. The high reduced chi squared value may be just in how I am propagating the uncertainty values, as the error bars in the T1 plot are very small in comparison with the rest of the graph.

## 5 Conclusion

In this experiment we used the technique of pulsed nuclear magnetic resonance to manipulate and make measurements of the bulk magnetization created by protons in glycerin, as well as calculating the gyromagnetic ratio of glycerin and teflon nuclei. We changed the magnetic field to change the angular frequency for which the nuclei will precess around the magnetic field. We tuned this magnetic field and RF pulses to find a resonant frequency such that the mixer will show that they are nearly the same. Measuring the magnetic field at this resonant frequency and using equation 4, we find the glycerin's nucleus (Hydrogen) gyromagnetic ratio in agreement with literature values and teflon's nucleus (Fluoride) gyromagnetic ratio nearly in agreement with literature values, but outside uncertainty.

Afterwards, we use RF pulses to rotate the bulk magnetization of glycerin in different directions to measure the spin-lattice ( $T_1$ ) and spin-spin ( $T_2$ ) relaxation time - a characteristic time describe how long it takes for non-equilibrium bulk magnetization in a certain direction to relax back to equilibrium. I find  $T_1 = 20.65 \pm 0.62$  and  $T_2 = 12.29 \pm 0.74$ , with a reduced chi squared of 5.21 and 1.33 respectively. Despite having relatively small uncertainty bars, there is still room for improvement. The uncertainty in voltage from the oscilloscope was eye-balled to around 0.2 Volts, but a more robust investigation can more clearly quantify this. When sending through pulses of specific angle, there can be a better method to see whether it works well. For example, a  $\pi/2$  rotation should maximize the signal, but this was also eye balled. Furthermore, resonance was set with eye-ball of whether the signal looks in resonance, but there can probably be more robust definitions of whether you are in resonance or not. Either way, the measurements point to reliable measurements for both gyromagnetic ratios and both characteristic times.