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Lab 7: Positron Emission Tomography

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May 23, 2022

1 Background

A positron is emitted from decaying radionuclides. This positron will travel within some material before interacting and binding with a free electron forming positronium. However since positronium is unstable, it will (most likely) decay to two photons that will move 180 degrees apart, each with half of the mass energy of the original positronium. We can then use a detector to detect the photons emitted from this process and reconstruct an image of the material with radionuclides.

There are two methods to try to reconstruct this image, we can either place the material and have many detectors take a look at the material, or we can have the material move and rotate with only 2 detectors. For a material with complex structure and many elements to it, like human tissue, we can use many detectors to create a highly resolved image. In the scope of this lab, we are at max taking a look at three radionuclide materials, so having many detectors is overdoing it. We can simply move and rotate on a platform to reconstruct an image of a couple sources with two detectors along some line of response.

In this lab, we take a look at a single source and paired sources to create some PET scan. With these, we can better calibrate and set spatial resolution to set up a process to create a detailed image of an unknown configuration.

2 Calibration and Setup

Firstly, we would like to ensure that the setup is working properly with the Photomultiplier Tubes (PMTs) as the detectors. If we place a source right in front of one of the PMTs, we should be able to detect photons and view them in an oscilloscope. Using the Red Pitaya software, we can expect to view a peak as a proxy of the photons, which is seen in Figure 1.

We would like to take a look at the spatial resolution of sources perpendicular to the Line of Response. To do this, we ran a single source through the PMT with different step sizes placed in the center of the plate. We then take a look at the first row of data (which in this case is simply counts) and plot it out to see how well defined a peak is for a given step size. Since this is a counting process, we estimate the uncertainty for each point as $1/\sqrt{N}$, where N is the amount of counts for a given measurement.

We can then look at a window near the center of the plate (where we expect the source to be), around 14 cm, and see how well defined the peaks are for a given step size. We should expect a smaller step size to more clearly define a Gaussian peak. However, that does not seem to be the case here. Looking at the counts as a function of their lateral distance perpendicular to the Line of Response, we see that decreasing the step size below 2.00 cm does not noticeably increase the 'peakiness' of the Gaussian peak.

Looking at the standard deviation for each fitted lateral step size in Table 2, there is not a significant change in how well defined the peak is. If anything, a higher step size performs a weaker Gaussian fit than lower step size based on the goodness-of-fit parameter of reduced chi squared oddly enough. In effect, we can safely set a lateral step size between 1.00 and 2.00 cm to resolve whether a source is in

Figure 1: Oscilloscope

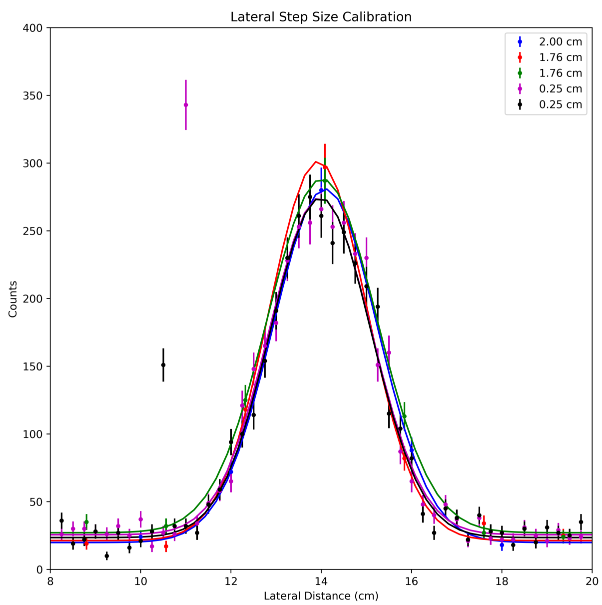

Figure 2: Calibration

Step Size (cm)	Standard Deviation (cm)	χ^2/N
2.00	1.16 ± 0.05	0.97
1.76	1.09 ± 0.05	7.54
1.76	1.21 ± 0.06	2.64
0.25	1.13 ± 0.02	338.26
0.25	1.12 ± 0.02	195.74

Table 1: Gaussian Fits

a certain location or not. Finer lateral steps will not produce a 'peakier' Gaussian distribution, so the ideas is that the source should not be even more defined when reconstructing the image.

3 PET Scan Data

We can now turn to the sinogram for different configurations.

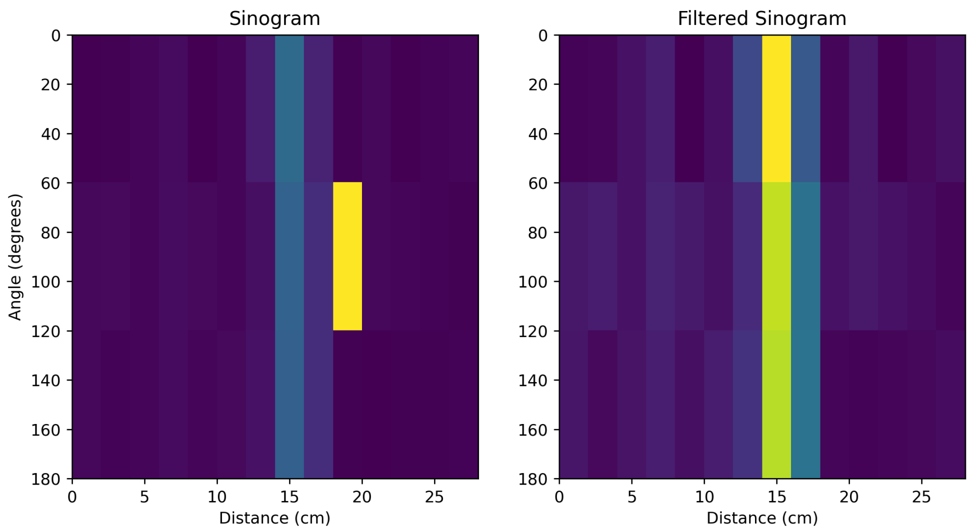

Figure 3: Sinogram - 1 Source

For a single source, we would expect to see one band that plots out the source's position. We decided to keep the source in the center, and we can see that behavior here in Figure 3. In this case, we chose 3 angles with an angle step of 90 degrees and a lateral step size of 2.00 cm. Each bin in which the PMT collects data was set for 10 seconds. We chose these very coarse, large bins simply as a starting point before delving further into the two source and unknown configuration. In effect, we didn't put too much effort into collecting fine data simply for one source in the center.

The data has only one large outlier, so we set that one bin value to the median of the entire data. The outlier is not expected and seems to be some error, so setting that data to the median of the data makes sense. We are not getting rid of the general idea of the figure. The actual image reconstruction is shown in the following section.

We can complete this for a paired source configuration and figure out whether we can find the path of the two sources.

Here, we left a source in the center of the platform and placed a source 5.6 ± 0.2 cm away from the central source, which is plotted out in Figure 4. We would expect a similar straight down source to Figure 3 accompanied by a source that is a sine wave that shows at max, it is 5.6 ± 0.2 cm away.

The way this sinogram was filtered was simply by taking the one outlier and setting its value as the median of the entire data, similar to that of the one source configuration for similar reasons. This should be able to drown out the noise from that one very high data. Other than that, the data looked well defined.

Looking at the sinogram in Figure 4, we see what we expect here. This specific configuration had 20 degree steps from 0 to 90 degrees. For N sources, we only need N+1 different angles, and we thought that 5 angles for 2 sources would be sufficient to make crude analyses. We were also hoping to gain a generally good sinogram for the 2 sources, and focus our attention on the unknown configuration, so again we do not use super fine steps. The lateral step size in this case was 1.5 cm, and the time for one measurement is 30 seconds. The parameters for this configuration and the previous 1 source sinogram were to investigate which specific parameters to vary before diving into the unknown configuration.

We noticed that we should probably increase the angle step size to be a bit finer, but the lateral

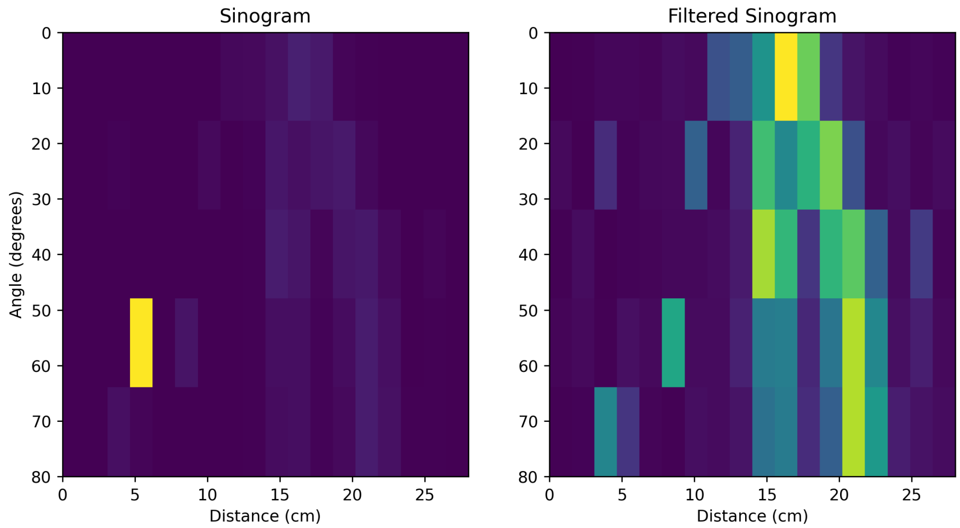

Figure 4: Sinogram 2 Sources

step size seemed sufficient. Again from Figure 2, we can see that lowering step size below 2.00 cm does not really increase resolution that much. From these sinograms, we notice that changing the time to collect data for one bin also does not drastically change the resolution of the sinogram. In effect, when setting the parameters to create a long detailed look at the unknown configuration, we set that to 10 seconds. We chose a lateral step size of 1.00 cm and an angular step size of 10 degrees from 0 to 180 degrees.

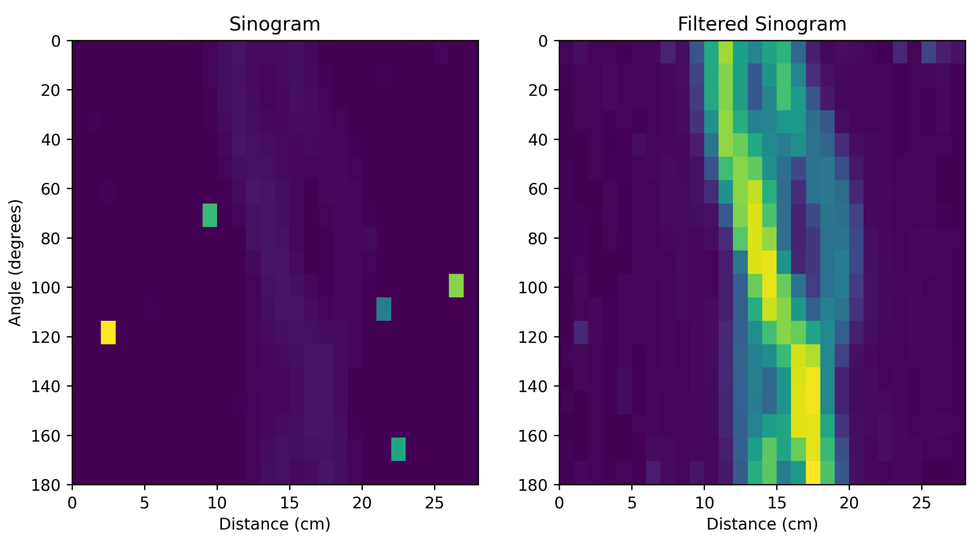

Figure 5: Sinogram Unknown Configuration

The way we filtered the data from the unknown configuration was by using a median filter of a box (1,3) in units of (x,y) pixels, where x is the lateral step bins and y is the angular step bins. Since the way to distinguish between sources is mainly found by dips and changes in x pixels, it is most convenient to make the filter along the y axis. This filter was able to get rid of high data points, which would be tough to do by eye and may bias the actual data. The previous filtering of putting the highest outlier to the median of the data did not work for this configuration.

Looking at the sinogram of the unknown configuration in Figure 5, we can see that there are more than just 1 source. Just by eye, it looks like there are either two or three sources. There is one that loops from about 10 to 20 cm reading from 0 degrees to 180 degrees. Likewise, there is another one that loops from 15 to 20 cm from 0 to 80 degrees. Since there is some count at 15 cm and 140 degrees as well as at 15 cm and 40 degrees, the sinogram also suggests that there may be a central source. However, we wish to spatially resolve their intensities and distances in an x-y plane rather than in a sinogram.

4 Tomographic Reconstruction

The sinograms look great, but they don't specifically show the sources' relative positions and relative intensities. What we then do is expand the data for each angle onto an $(N \times N)$ matrix of, where N is the amount of steps taken. For the first data collection run, the first column has the first angle's histogram. Each subsequent column has the same data but is extended to have that $(N \times N)$matrix shape. We then compute each subsequent $(N \times N)$ matrix for each angle step. For each angle, we then rotate by that angle using scipy's ndimage rotate method. The matrix is rotated in the plane defined by the two axes using spline interpolation of order 1. To make a final image from the sinogram to position, we then take a composite of each rotated matrix, which is done by multiplying the counts from each rotation. We then normalize the data by taking each data point to the 1/m power, where m is the amount of angular bins there are.

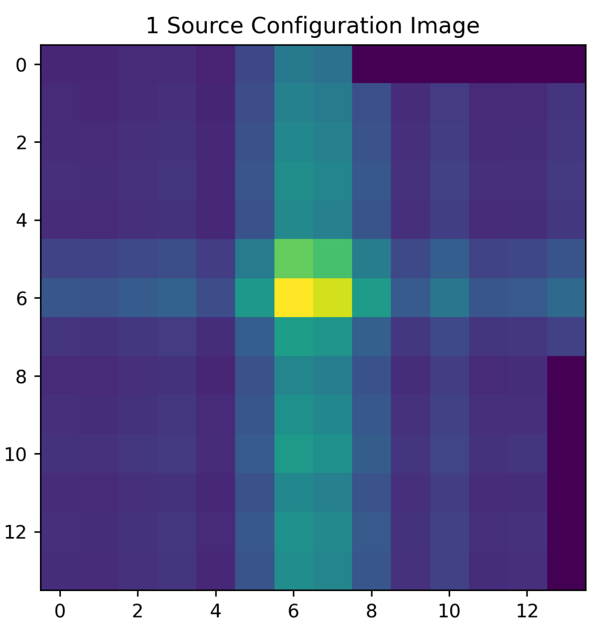

Figure 6: Reconstructed Image 1 Source

From this stacked final image, we can then assume that sources spread out radially independent of angle. To start looking at the relative intensities, we can fit Gaussians along a given axis, and use the fitted amplitude to compare against different sources as a proxy for intensity. Assuming the fitted source values would give Gaussian distributions the same in the x and y axis, I chose the y axis to get rid of the bleeding counts from nearby sources which is apparent in both the two source Figure 7 and unknown source configuration Figure 10.

To define a source, I found a highly concentrated pixel by eye, and made a 5 by 5 pixel square which has the highest pixel count at the center. From there, I fit along the y axis of the highest count x column to a Gaussian. I settled for defining a source within a 5 by 5 pixel square after taking a look at the reconstructed image, and finding how many pixels would best define one source with little bleed over between sources. This was an iterative process that took a couple tries to best define the region of a source using the Gaussian fits. A 7 by 7 pixel square would have too much bleed over from nearby sources, and a 3 by 3 pixel square would not have enough data to make any meaningful conclusions. Further, to account for any background noise which may be present, the fit is done on a Gaussian distribution with a constant.

This is completed for both the 2 source and unknown source configuration. For the one source configuration, the image reconstructed is not fit using any Gaussians because these are used to compare relative intensities, but there's no other source to compare against. Either way, the reconstructed image is shown in Figure 6.

For the 2 source configuration, we see that the source in the center and the one a couple centimeters away are apparent in the reconstructed image.

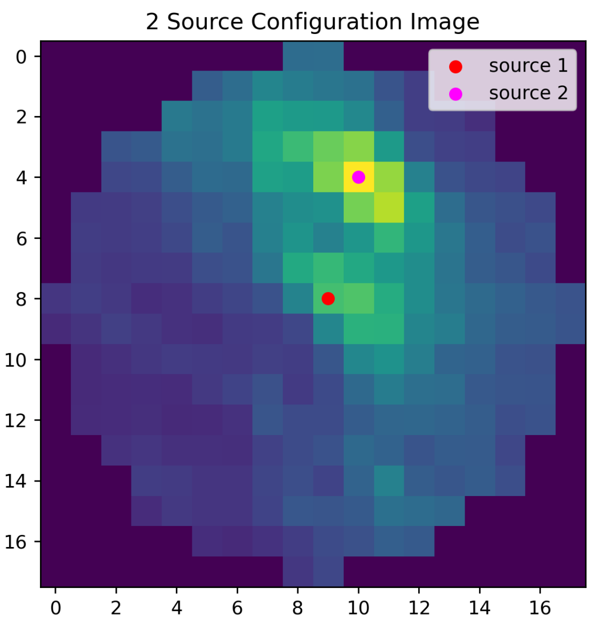

Figure 7: Reconstructed Image 2 Sources

We take the intensity as the fitted amplitude and find it to be with quite large error bars. The ratio for the relative intensity of source 1 to source 2 is 0.71 ± 0.27. The distance between them is calculated as the Euclidean 2-D distance in pixels from what was seen as the center of the source. Further, we can use the fact that each pixel is 1.5 cm to convert to centimeters. We also estimate the uncertainty for each coordinate as half of a pixel, and use the following equations to propagate uncertainties.

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \tag{1}$$

$$\Delta D = \sqrt{\frac{\partial D}{\partial x_1} + \frac{\partial D}{\partial x_2} + \frac{\partial D}{\partial y_1} + \frac{\partial D}{\partial y_2}}$$

$$\Delta D = \frac{1}{D}\sqrt{((x_1 - x_2)\Delta x_1)^2 + (-(x_1 - x_2)\Delta x_2)^2 + ((y_1 - y_2)\Delta y_1)^2 + (-(y_1 - y_2)\Delta y_2)^2} \tag{2}$$

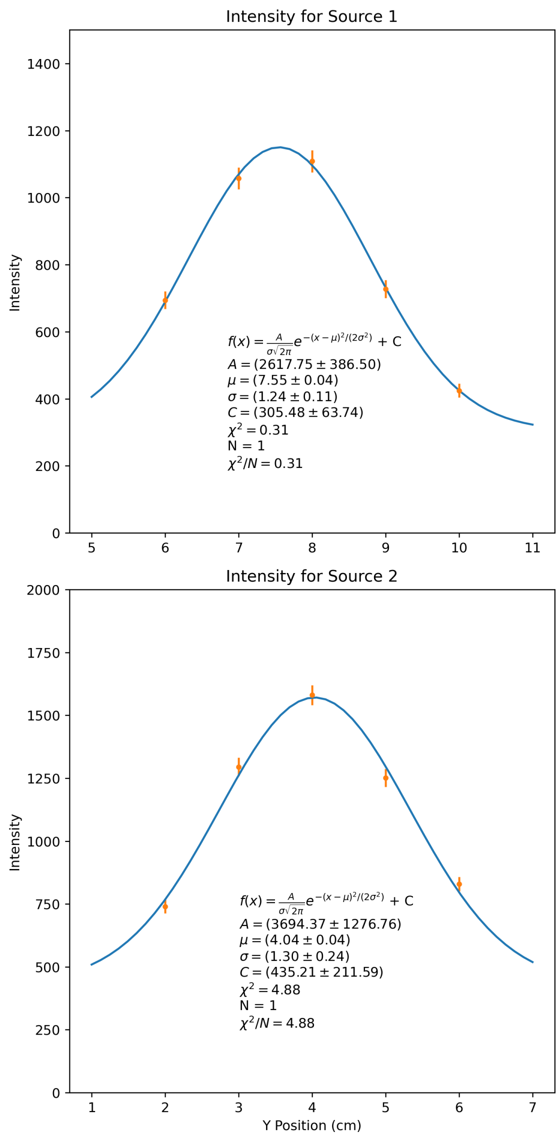

Figure 8: Gaussian Fits 2 Sources

The calculated distance is then 6.2 ± 1.1 cm.

For the unknown configuration in Figure 10, we can see that there are multiple sources appearing.

We complete a similar analysis for the unknown configuration and find the intensity ratio of source 1 to source 2 is 0.78 ± 2.17, the ratio of source 2 to source 3 is 6.01 ± 15.63, and the ratio of source 1 to source 3 is 4.66 ± 6.62. Likewise, the distance between source 1 and source 2 is 6.40 ± 0.71 cm, between source 2 and source 3 is 4.00 ± 0.71 cm, and between source 1 and source 3 is 5.00 ± 0.71 cm.

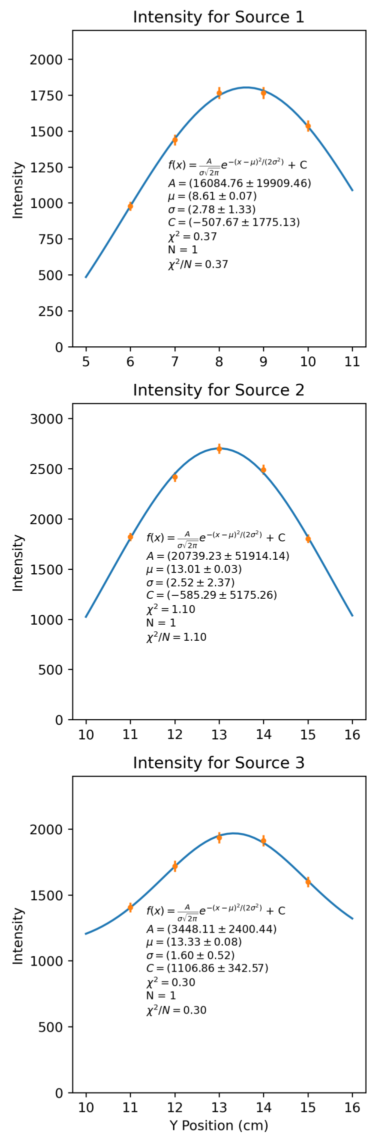

Figure 9: Gaussian Fits Unknown Configuration

5 Conclusion

As can be seen from the massive uncertainty for different values as well as the Gaussian fits performed, these values are not to be trusted that much. We see that the idea of finding out how 'peaky' a Gaussian peak is does not really translate well into being able to spatially resolve many sources. A lot of the uncertainty can stem from simply seeing that our image was not created with enough resolution to make any reasonable conclusions.

Using only 5 angles with 1.5 cm lateral step sizes, the doubly sourced image is not well defined, and using any other filtering process will reduce the data too much so as to have little meaning. I tried using a median filter (as was done for the unknown configuration) for the two source configuration, but the image was too noisy to discern what was a source and what wasn't.

Another way to test this method of choosing step sizes and Gaussian fitting to find relative inten-

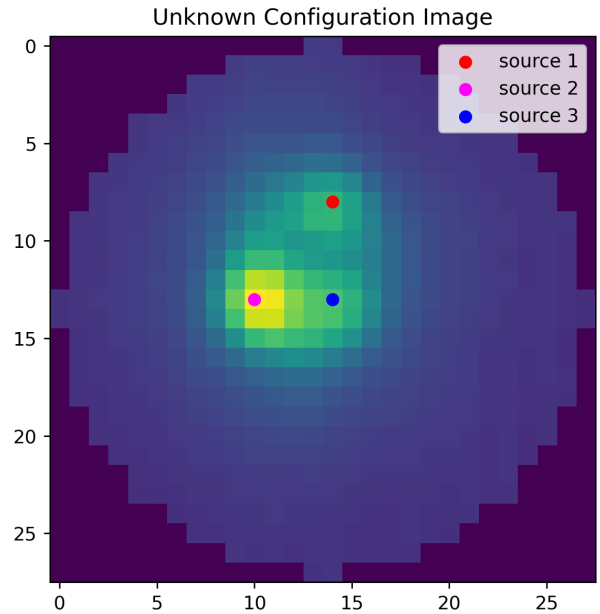

Figure 10: Reconstructed Image Unknown Configuration

sities is to compare the calculated values from the two source configuration to measured values in the lab. In the lab, we find that the sources are of the same element and energy, such that their relative intensities should be 1.00, but it is calculated as 0.71 ± 0.27. Likewise, the distance measured between the sources was 5.6 ± 0.2 cm, while the calculated distance was 6.2 ± 1.1 cm. Both of these calculated values do not agree with what we expect from measurement, which is an indicator that our results for the unknown configuration cannot be made with much confidence.

Simply taking a look at our values for the calculated relative intensities between sources in the unknown configuration shows how untrustworthy our results were. Each ratio has uncertainties greater than the value itself, so it really does not make much sense without noticing the flaws in our measurements and analysis. The relative distances, however, kind of make sense with regards to how they look in the reconstructed image. But the fact that the distance was not accurately reproduced for the two source configuration means that we cannot make confident claims about either of the relative intensity or the distances.

Likewise, the fit was made with a Gaussian distribution and a constant. If this method had worked well, then the constant should either be near zero, or at least have the same value over the same data. The 2 source configuration has very high constant value, implying lots of noise in the background, while the unknown configuration has negative constant values and is not the same within all three fits. Having the data not fit the Gaussian distribution shows a flaw in measurement and analysis. There has to be finer step sizes to be able to have more data to clearly define a Gaussian distribution.

In the future, we can take the fact that the sources are sized about a centimeter in diameter to know that we needed finer step sizes. With finer step sizes, we would have a more spatially resolved reconstructed image of both the unknown and two source configurations. I believe that the amount of angles used for the unknown configuration is enough to uniquely define each source, but more would not hurt. Many more angles would have allowed the two source configurations to have more well defined positions for the sources. With a more resolved final stacked image, we would have more data and less obvious bleedover. With this, we would have more than 5 data to fit onto a Gaussian distribution, which should decrease the uncertainty for our results.

To conclude, we find a relative intensity for the two source configurations of 0.71 ± 0.27, when we should expect something near 1.00. Likewise, the calculated distance between the two sources was 6.1 ± 1.1 cm, when we expect 5.2 ± 0.2 cm. Likewise for unknown configurations, relative intensities between sources were 0.78 ± 2.17, 6.01 ± 15.63, and 4.66 ± 6.62. The relative distances were 6.40 ± 0.71 cm, 4.00 ± 0.71 cm, and 5.00 ± 0.71 cm. Our results, with large uncertainties, are untrustworthy, and we can use finer step sizes in the future as well as smaller angle steps to more clearly spatially resolve the positions of both configurations.