Lab 2: Drop Pinch-Off

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1 Setup/Overview

This experiment aims to detect a relationship for the pinch-off of a drop with a couple different physical properties. With a high-speed camera, we can look at how the drop evolves over time, and which physical property may dominate the pinch-off at different stages of this evolution. To complete this, we record a drop falling and measure the minimum radius at times before the pinch off occurs. The potential parameters that could describe how the minimum neck radius may evolve over time are the following, along with appropriate dimensions mass (M), length (L), and time (T).

Parameters	**Dimensions**
Neck Radius r_{min}	L
Time from Pinch-Off τ	T
Fluid Density ρ	ML^{-3}
Surface Tension γ	MT^{-2}
Viscosity η	$ML^{-1}T^{-1}$

Table 1: Parameters used to complete dimensional analysis.

Using these parameters, to accurately describe the evolution of the drop over time, we completed some light dimensional analysis to find an equation that would describe the neck radius with a function of time as well as a function of the other physical parameters:

$$R_{min} = F(\tau)G(\rho, \gamma, \eta) \tag{1}$$

The goal of this experiment is to find the general exponent for $F(\tau)$ that would accurately describe what was found in the lab. After finding some scaling laws that make sense, we can then fit regions of the data of r_{min} measured to a power law, like this equation:

$$R_{min} = Ax^b \tag{2}$$

To capture videos of the fluid's drop, we used a high speed camera camera. To analyze footage of the drop, the ImageJ software was used.

2 Data

2.1 Scope of Experiment

It is important to note that this experiment creates a relationship between the minimum radius and two functions, one of time and the other a function of density, surface tension, and viscosity. There are many other physical parameters that can affect what we measure, such as the temperature of the liquid.

Other systematic uncertainties that can be noted are things that create the setup, like the physical conditions (temperature, humidity, etc.) of the room that can affect any conclusions. Likewise, the pipette from which the liquid drops can also affect the shape and evolution of the drop. If the pipette were to be from a different type of plastic or has a different shape at the end, the evolution of the drop can possibly change.

To help mitigate some of these circumstances, we attempted to leave the room as it was when we started taking data. Either way, it is important to note.

Another limitation of this experiment is that we test only one liquid. All of the following data and conclusions can be drawn from the specific water glycerin mixture that was provided to us (we were not provided the specific density, viscosity, and surface tension presumably because of the difficulty of measuring the characteristics and also to help simplify the lab). Knowing that the scaling laws that were found have consistent units, the relationships found in this experiment can be replicated for another fluid, simply shifted by the specific values of that other fluid's physical properties.

Using the parameters in Table 1, there are three main time dependent relationships that were found:

$$R_{min} \propto (\frac{\gamma}{\rho})^{1/3}\tau^{2/3} \tag{3}$$

$$R_{min} \propto \frac{\gamma}{\eta}\tau \tag{4}$$

$$R_{min} \propto (\frac{\eta}{\rho})^{1/2}\tau^{1/2} \tag{5}$$

These relationships kind of make sense when describing how a drop makes the pinch-off behavior. First, the drop is dominated by surface tension keeping the liquid drop together against the viscosity, so the viscosity shouldn't be that important. This dominance is seen in equation 3. Once the drop starts to stretch, the viscosity and surface tension are competing with each other. The viscosity has the fluid wanting to move down, and the surface tension is trying to keep the fluid together, which can be noted in equation 4. Lastly, the drop is stretching thinly and is no longer being kept together by surface tension, so it should not really factor in this region, as can be seen in equation 5.

From the three proportionalities, we have 3 different relationships between time and the minimum radius, which should evolve over time: $T^{2/3} \rightarrow T^1 \rightarrow T^{1/2}$

2.2 Data Handling

For each video of different drops that was recorded, there was a general procedure that was followed to ensure that the experiment flowed well.

First, we would insert the SD card into the camera and turn it on. Depending on the data that we already had, we would discuss how zoomed in and what region of the drop we wanted to analyze. From this discussion, we would set a specific resolution and frame rate, and start this "run". Using lots of trial and error, we would tweak the focus, aperture, and position of the camera to get the drop/pipette in focus. Then we would place the ruler in the frame and record a short video, which would be saved onto the SD card, and imported into the computer to start some image calibration. In order to use ImageJ, we had to use the ffmpeg software to convert the mp4 video to an avi file. After dragging the file onto our directory, we would eject the disk and put it back into the camera. Then one of us would look at the calibration video and start making a couple of measurements using ImageJ of the amount of pixels per millimeter tick mark. Using the frame rate and FPS already set, the other person would make sure everything was in frame, well lit, and correctly focused to start recording. Untwisting a clamp on the pipette which was used to keep the liquid from continually pouring, the liquid would then create a drop and we would record the pinch-off. After recording a couple of drops, the same person would save the video onto the disk, import it into the computer, and convert into an avi file so that we could start taking measurements of the minimum radius. This would end one run.

To change the aperture and let in more light into the detector, we would rotate the blue ring. In order to have a video in focus, we would first focus using the lens with the yellow ring. Using the focus aid software on the camera, we would then translate the camera with minute movements in order to have granular changes in focus and to make sure the drop was in frame. This way we mitigated lots of blurriness that could affect the confidence in our minimum radius measurements. Furthermore, to make sure that the image calibration would provide accurate conversion measurements, many measurements were taken when taking data for the number of pixels per tick mark.

After completing the entire experiment, we used 3 different runs with the following information:

Specific Run	**Resolution (pixels, pixels)**	**Frames per second (Hz)**
Run 1	640, 240	8819
Run 2	1280, 720	1502
Run 3	336, 96	38565

Table 2: The specific runs we ran.

Using the ImageJ, the process of finding the minimum radius was to simply look at where there is a minimum in the radius length. The data was initially taken with this very arbitrary procedure, then a jupyter notebook was shared that uses the python package cv2. This library has many helpful functions to help in computer vision analysis. This helps measure the radius on a frame-by-frame basis by computing the contour of the fluid's drop and tracking the minimum of the vertical axis of this image over time.

After completing the data taking using the jupyter notebook, the txt files describing the neck's width would be saved for each drop.

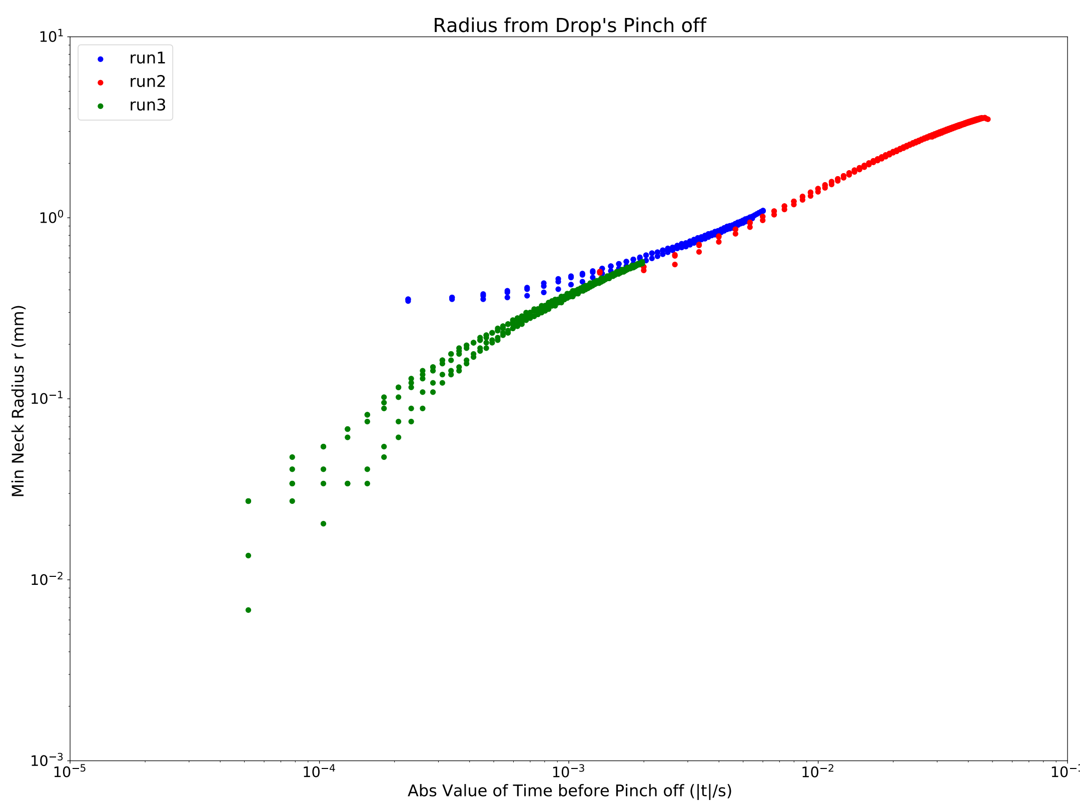

Figure 1: Different Runs Data

Now that we have data that describes the width of the actual neck, now we can start normalizing everything by converting from pixels to millimeters and frames to seconds. By converting to actual length and time, we would be able to compare the data from different runs between each other. To complete the conversion from pixels counted to a length measurement, the following two equations were used.

$$R(R_p, P) = \frac{R_p}{P} \tag{6}$$

$$T(T_f, F) = \frac{T_f}{F} \tag{7}$$

R_p is radius in pixels and T_f is time in frames. P is pixels per mm. F is frames per second. Using these equations, I was able to describe the drop's evolution in all three runs in terms of length and unit time. The frames is such that when there is no more contact between the fluid at the top and the drop falling, then the next frame is noted as the time that there is pinch-off. This is indicated as t=0 in the following figures. The radius starts at a large value and decreases as the time gets closer to pinch-off. In effect, a good way to read the following plots is that the evolution of the drop comes from the right and evolves to the left.

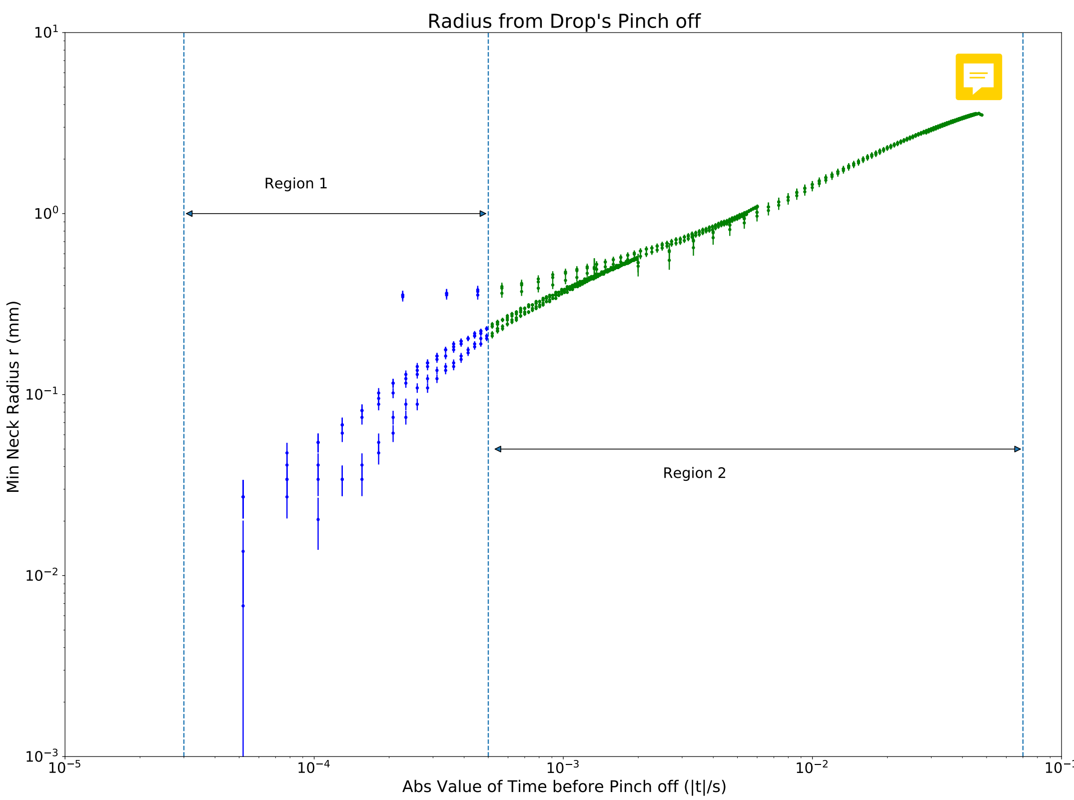

Figure 2: Two Regions Identified

2.3 Plotting Data

From Figure 1, the data is shown and each color represents a certain run that was taken. As is a bit expected, data in the same run follows a very similar correlation in comparison to other data in the same time regime. To ensure that when scaling this data to a power law of type equation 2 the units worked out correctly, I did not want to have an x value - here corresponding to time - that would be raised to a weird power. So instead, I decided to make it dimensionless by normalizing all the time by 1 second.

From Figures 1 and 3 it is easy to notice that there would be more interesting data to be found in the lower time, where the drop is closer to pinch-off. The data does not support one universal power law to all data, but interesting behavior can be seen throughout. There is also quite a mild transition from one power law in the leftmost regime, about $|t| < 8 \cdot 10^{-4}s$, and the rightmost time regime of $|t| > 3 \cdot 10^{-3}s$. By having data in millimeters and seconds, it's also important to see that the time scales that this experiment is going through are extremely small, in the magnitudes of milliseconds. With an even more powerful camera that has a higher frame rate count, there could be more interesting data to be developed in the lower time regimes.

2.4 Power Law Fits

The regions I looked at were chosen to best represent the fit near that regime. From Figure 2, it's clear that we have two different regions that has a very soft transition between the power laws that may best describe the data. Initially, I planned to discard a certain transition area between where it goes from linear on the smaller time to the smaller power law in the right, but it seemed too arbitrary to discard lots of data. Instead, I decided to include all of the data and see what sort of conclusions can be drawn, despite having a soft transition. By including this, it may affect the fits, but it properly describes what was measured. When deciding on where to exactly place the boundary between the two time regions, I looked at where the linear scale starts dropping off. Also, when completing preliminary data analysis, I would try to tweak the boundary to have a reduced chi squared, our "goodness of fit" parameter, to be around 1. After completing this analysis, I decided on making a boundary at $t_{bnd} = 5 \cdot 10^{-4}s$.

The time regime with smaller time than the boundary, $|t| < t_{bnd}$, is described here as Region 1. Region 1 describes the neck's radius closer to pinch-off. Region 2 is the time regime with time greater than the boundary t_{bnd}, which describes the early evolution of the droplet.

With a linear scale in the lower part, we can expect that the model spectrum relationship of $T^{1/2}$ does not really apply here, as it would be noticed at a smaller time, closer to pinch-off. This also gives rise that the expected power law to describe the larger time (right side on figures 1 and 3) before the pinch-off happens is probably the $T^{2/3}$ power as described in equation 3.

To make sure that there is sufficient data to complete a proper fit in a time regime, I added a condition that a drop's time regime must have at least 5 data points in the regime. This is a bit arbitrary, but allows us to not extrapolate from the few data that strays from Run 1 into Region 1.

Using this distinction, Figure 1 shows that there should only be data from Run 3 that is fitted onto Region 1. The plots in Figure 3 verify this, and provide the power at which these 5 drops can be described by time. Looking at the plots, many are close to 1. An important property from these fits is the reduced chi squared. Most of the values are near 1, with only 1 being over 1, which is what is expected from equation 4. This is a bit deceiving because usually, when a reduced chi squared is much less than 1, the error bars are expected to be overestimated. However, most of the error bars are already small. This can be attributed to the fact that the plot is in log-scale, so errors that are large in a greater time value may seem very small in comparison to the rest of the plot, but it is still there. Either way, this indicates that there is the availability of a more robust error analysis for this experiment.

Since all of the drops had more than 5 data points at times greater than t_{bnd}, then all 11 drops for the 3 runs were fitted to some power law in Region 2, which can be seen in figures 4, 5, and 6. All

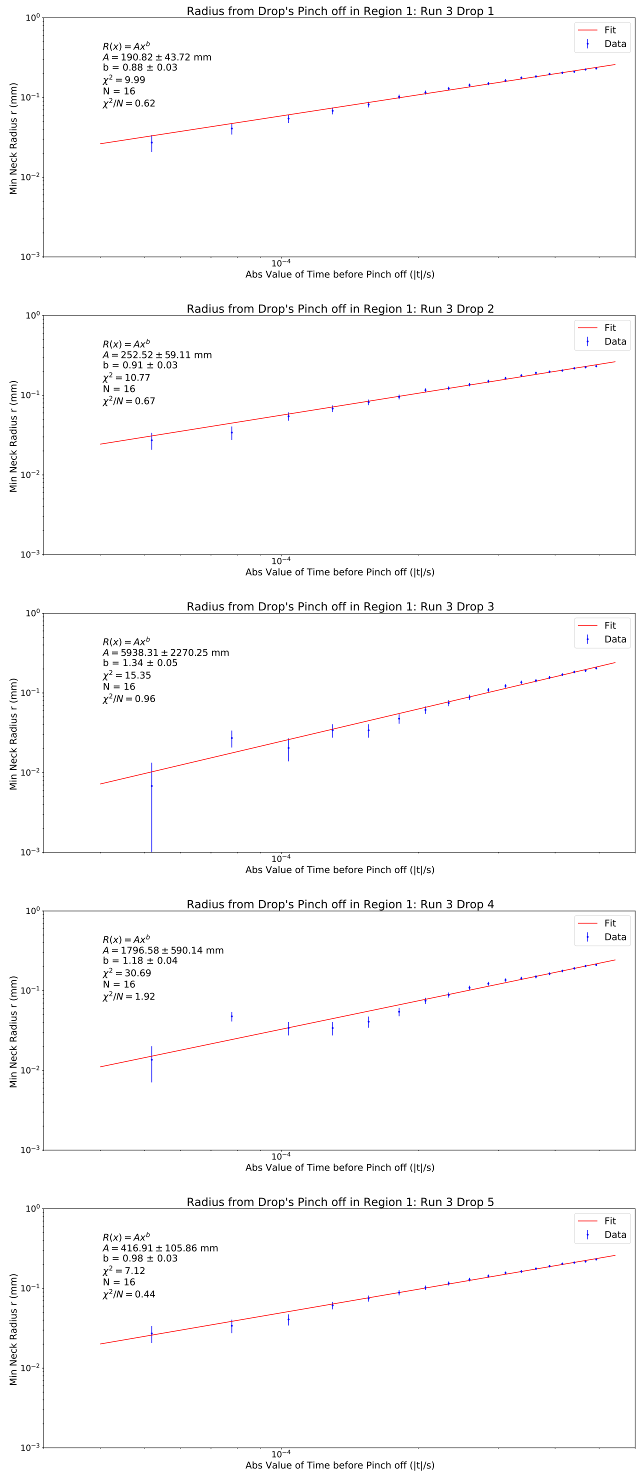

Figure 3: Drops Fit in Region 1

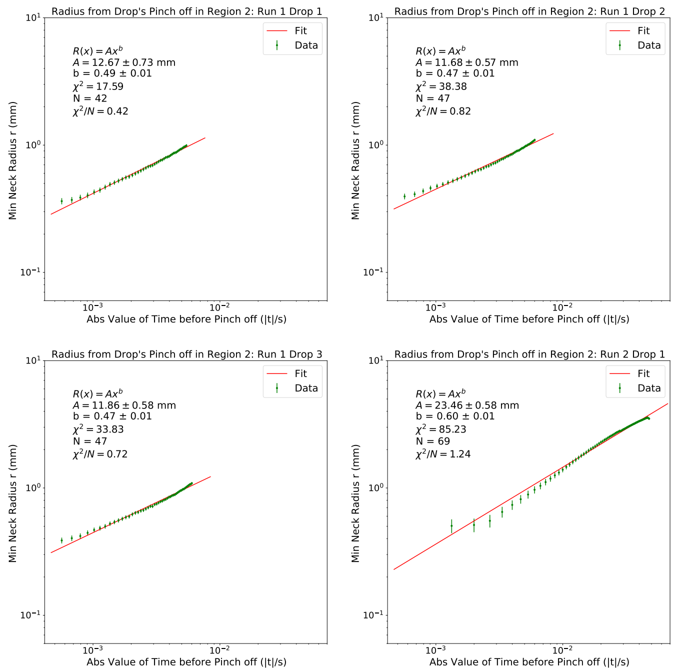

Figure 4: Drops Fit in Region 2. Part 1

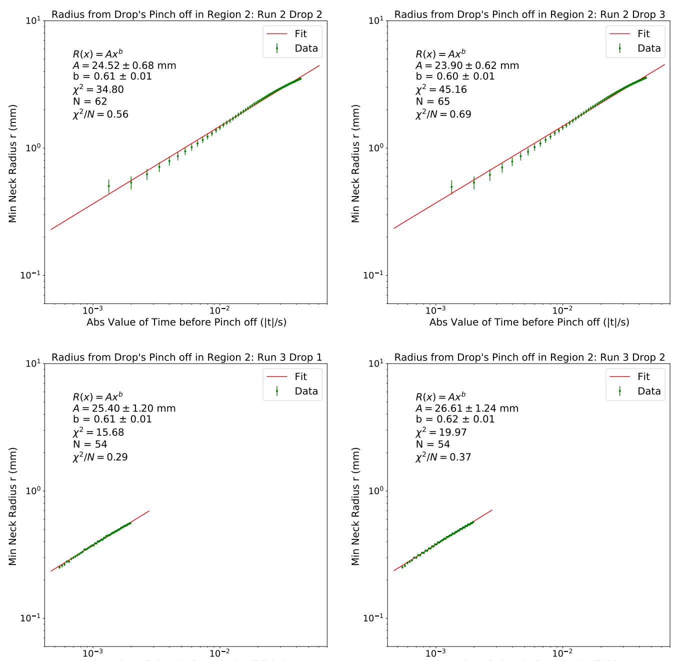

Figure 5: Drops Fit in Region 2. Part 2

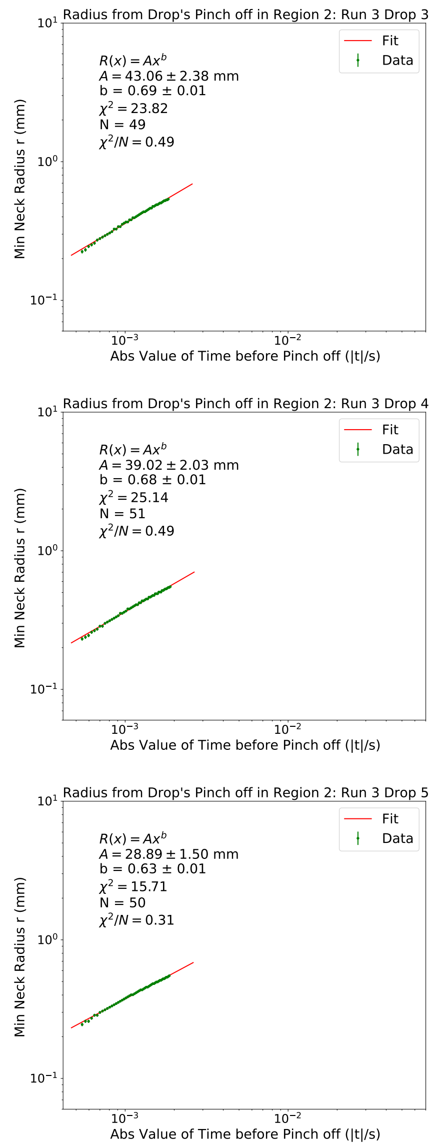

Figure 6: Drops Fit in Region 2. Part 3

of the exponent fits are in the general area of 1/2 to 2/3 as the scaling law. Lots of the fits actually underestimate what the fit should actually be. Something that could account for this greater neck radius at time closer to 0, is the software package cv2 may be looking at some fuzziness in the video when there is pinch-off that makes the software overestimate the actual radius. The software may be overestimating what the actual radius is.

Despite being within an order of magnitude of 1, the reduced chi squared for all of the fits in Region 2, just like Region 1, deviates a bit from 1. This is another indication that my error analysis may not be robust enough to draw valuable conclusions.

3 Discussion

3.1 Uncertainty Analysis

The two equations used to convert from frames and pixels to seconds and millimeters, equations 6 and 7, have some sort of uncertainty within both of them. For the conversion from frames to seconds, the frames per second is such that the camera provides this number. Likewise, we can choose which frame the pinch-off occurs, where there is no more connection of the drop with the liquid. These two variables have such small uncertainties within them, that the main uncertainty that was analysed was the uncertainty in the neck's width measured. The equation that was used to calculate the millimeters of a neck's width is a function of the pixels per millimeter and the amount of pixels measured.

To calculate the uncertainty in one of these measurements, we can use the general formula for a function of independent variables, like in the following equation:

$$\frac{dR}{R} = \sqrt{(\frac{dR_p}{R_p})^2 + (\frac{dP}{P})^2} \tag{8}$$

$$\frac{dR_i}{R_i} = \sqrt{(\frac{dR_{p,i}}{R_{p,i}})^2 + (\frac{dP_i}{P_i})^2} \tag{9}$$

Here, the index i simply identifies the specific drop for which the uncertainty was calculated for. In this equation, dR_p is the uncertainty of pixels we measured when using ImageJ at first, and later was extended upon the cv2 python package. dP is the uncertainty in the pixels per mm measurement when completing calibration of the camera. The camera was zoomed in so much that the tick marks for a mm would be much too small to see.

To measure dP, we take many measurements in between tick marks at different millimeter intervals. The dP is the standard deviation of these measurements per millimeter, and the P is simply the mean. The units of dP and P are just number of pixels per millimeter.

Since the measurement of R_p is made along an axis perpendicular of where the fluid is dropping, the uncertainty should be related to the amount of pixels in that axis. The greater the axis, the more likely we are to make small deviations to affect the greater radius measured. A smaller axis would have a smaller uncertainty because there less amount of pixels to choose which would be incorrect. Despite being computed by the cv2 python package, there should still be some uncertainty. We estimate dR_p as the 1% of this axis, which can be taken as 1% of l, where w x l defines the resolution of the video taken. The 1% was chosen to best create a reduced chi squared value. This number was chosen also because when we would take the standard deviation of the pixels when we used ImageJ, it would be within a pixel of this value. The units of R_p and dR_p is only pixels. Because there is not significance to less than a pixel, there is a floor function, where any uncertainty dR_p less than one has uncertainty of 1 pixel. From these calculated dR_p and dP, we could find some error bars for each drop's measurement.

Despite my reasoning, this does not cover the entirety of all of the uncertainties that go along with the measurements. The dR_p is a bit generic and ambiguous. I have tried my best to optimize for a reduced chi squared around 1, but there may be better ways to do this. When creating measurements of P, the pixels per millimeter, the tick marks are quite large. We tried our best to measure at the middle of these tick marks, but this uncertainty may be better optimized. Also, with a floor of 1 pixel for dR_p, then there is only so much uncertainty that can be confidently analyzed with the runs that were setup.

A way to help mitigate these uncertainties could be with the camera. Despite being a very powerful camera, there may be other cameras that could provide a larger CCD sensor. The zooming in with a more powerful camera could have the pixels per millimeter much greater than currently. Somebody completing a similar analysis with this more powerful camera may have the floor of a pixel uncertainty at a much more zoomed in scale than used in this experiment. Furthermore, the camera does bring with it a bit of uncertainty. The main one that can probably be noticed is the blurriness when not completing a well enough focus. With an out of focus frame, the cv2 software might think that the blurriness to be part of the minimum neck radius, increasing the neck radius. Without more data its tough to say with lots of confidence, but this seems to be what happened here. When looking at the Region 2 fits, which can be seen in Table 4, the values closer to the expected 2/3 come from Run 3, which was the final setup we ran. The second close fits are those from Run 2, and the worst ones are from Run 1. This may show that there is some systematic error that was not properly accounted for in the first day of taking data. This indicates that we focused to a greater precision as we grew more familiar with the camera. There is not a solid method to help mitigate this other than just having had more experience. With limited lab time, though, this is tough to do.

Specific Run, Drop	Fit Power Law	Expected Power Law
Run 3, Drop 1	0.88±0.03	1.00
Run 3, Drop 2	0.91±0.03	1.00
Run 3, Drop 3	1.34±0.05	1.00
Run 3, Drop 5	1.18±0.04	1.00
Run 3, Drop 4	0.98±0.03	1.00

Table 3: Fits in Region 1, with Expected Power law of T^1.

3.2 Comparison with Literature

Here, we take the initial 3 relationships that were found from dimensional analysis as what we would expect the data to follow. However, we drop the relationship of $T^{1/2}$ because of the range at which we captured our data. For Region 1, there is only 1 of the 5 drops that lands on the expected value given its error bars. All others are very near the value of 1, as can be seen in table 3. For fits in Region 2, which can be seen in table 4, only 1 of the data lands within error bars of the expected value of 2/3. Again, this may be because the error bars are incorrect in and of themselves. However, most values are between 0.6 and 0.7, indicating that it is very near to being the expected value. There are no error bars in the expected values, because the dimensional analysis provides us with one number, and there are not any uncertainties that go with what is expected.

Specific Run, Drop	Fit Power Law	Expected Power Law
Run 1, Drop 1	0.49±0.01	0.67
Run 1, Drop 2	0.47±0.01	0.67
Run 1, Drop 3	0.47±0.01	0.67
Run 2, Drop 1	0.60±0.01	0.67
Run 2, Drop 2	0.61±0.01	0.67
Run 2, Drop 3	0.60±0.01	0.67
Run 3, Drop 1	0.61±0.01	0.67
Run 3, Drop 2	0.62±0.01	0.67
Run 3, Drop 3	0.69±0.01	0.67
Run 3, Drop 4	0.68±0.01	0.67
Run 3, Drop 5	0.63±0.01	0.67

Table 4: Fits in Region 2, with Expected Power law of $T^{2/3}$.

There are many different reasons why the there is only 2 of the 16 fits that fall within error bars of the expected value. There simply is not too much data taken at either end of the scales. Furthermore, a very soft transition point from T^1 and $T^{2/3}$ indicates that this may be with how the data was collected and manipulated. We expected a more sharp transition point when speaking with lab instructors. As mentioned before, the blurriness and focus, despite having done our best to mitigate any errors, could be slightly off changing what the package cv2 analysis as the neck radius. Also, speaking with the lab instructors about the experiment, we found that the fluid was not as viscous as expected, and did not have much glycerin in it. In effect, the liquid did not stretch much, which may effect how the drop would evolve.

Despite having values that are near the expected values, the data points do not fall within the error bars of the expected values. In effect, we cannot make adequate conclusions from the data with the uncertainty analysis completed.

3.3 Future Steps

Looking back at the procedure, data analysis, and the overall experiment there are a couple things that could be improved upon. We could take more data in the smaller time regime to try to find the third time region with time scaling at $T^{1/2}$. The jupyter notebook used to compute the minimum neck widths may change if we used different methods to complete the contour. Furthermore, a more thorough analysis of the uncertainties at play may allow for more well calibrated data. Also, different fluids may affect the measured values, so in the future, we can complete similar measurements for different rho, eta, gamma. If the time dependence is true, then it should hold for many liquids. Another thing that could be optimized, is by using a ruler that is much more accurate than the millimeter one used. We could use a micrometer paper for a future experiment extension.