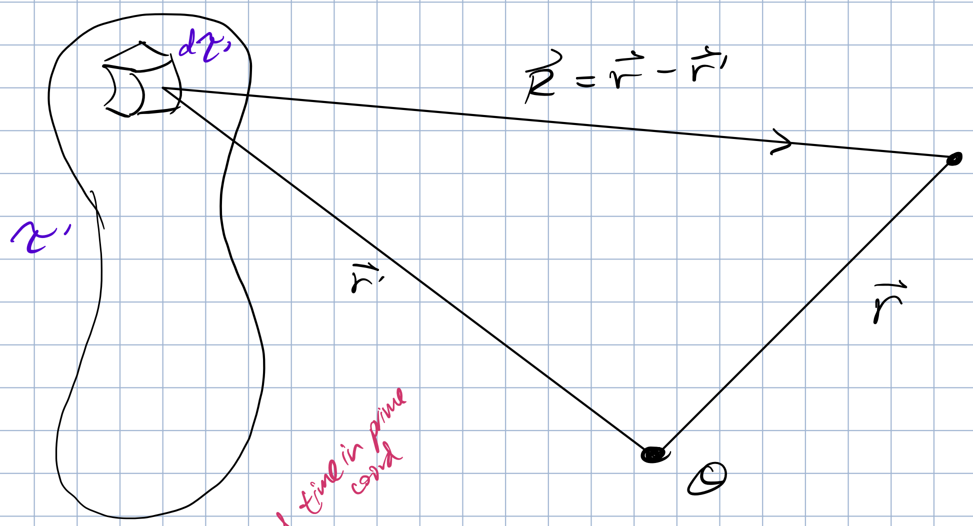
Midterm tings :(

$\Box \equiv \nabla^2 - \frac{1}{c^2}\partial_t^2$

find $A(\vec{r}, t_r)$ $\quad \varphi(\vec{r}, t_r)$



time in prime coord

$$t_{delay} = |\vec{R}|/c$$

$$t_{ret} = t' = t - t_{delay} = t - |\vec{R}|/c$$

$\varphi(\vec{R}, t)$

$$\nabla^2\varphi = \frac{1}{R^2}\partial_R(R^2\partial_R\varphi)$$

$$\frac{1}{R^2}\partial_R(R^2\partial_R\varphi) - \frac{1}{c^2}\partial_t^2\varphi = 0$$

$$\varphi(R,t) = \frac{\chi(R,t)}{R}$$

$$\frac{1}{R}\partial_R^2(\chi) - \frac{1}{Rc^2}\partial_t^2(\chi) = 0$$

$$\partial_R^2(\chi) - \frac{1}{c^2}\partial_t^2(\chi) = 0$$

sol^{ns}: $\chi(R,t) = f(R-ct) + g(R+ct)$

$$\chi(R,t) = f'(t - R/c) + g(t - R/c)$$

Green's f^n

$$\nabla^2\varphi(\vec{r},t) - \frac{1}{c^2}\partial_t\varphi(\vec{r},t) = -\underline{f(\vec{r},t)}$$

known source distribution

$$\varphi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_V \frac{f(\vec{r}')}{|\vec{r}-\vec{r}'|}d\tau' \longrightarrow \nabla^2\varphi(\vec{r}) = \frac{f}{\epsilon_0}$$

def^n of dirac delta : sol^n to Poisson w/ spatial dependence $\rightarrow \nabla^2\left(\frac{1}{|\vec{R}|}\right) = -4\pi\,\delta(|\vec{R}|)$

Green's f^{n}s

3D dirac impulse f^n

structure & generalize

$$\nabla^2 G(\vec{R}) = -\delta(\vec{R})$$

$$G(\vec{R}) = \frac{1}{4\pi}\frac{1}{|\vec{r}-\vec{r}'|}$$

known source distribution

$$\mathcal{L}\,\Psi(\vec{r}) = -f(\vec{r})$$

$$\mathcal{L}\,G(\vec{r},\vec{r}') = -\delta(\vec{r}-\vec{r}')$$

operator

find $G(\vec{R})$ w/ impulse & $f(\vec{r})$ $\rightarrow$ full f^n

$$\Psi(\vec{r}) = \int_{V'} G(\vec{r},\vec{r}')\,f(\vec{r}')\,d\tau'$$

Corollary of Gauss

$$\vec{\nabla}(\Psi\,\vec{\nabla}G) = \Psi\nabla^2 G + (\vec{\nabla}\Psi)(\vec{\nabla}G)$$

$$\vec{\nabla}(G\,\vec{\nabla}\Psi) = G\nabla^2\Psi + (\vec{\nabla}G)(\vec{\nabla}\Psi)$$

$$\Psi\nabla^2 G - G\nabla^2\Psi = \vec{\nabla}(\Psi\,\vec{\nabla}G) - \vec{\nabla}(G\,\vec{\nabla}\Psi)$$

$$\int_V (\Psi\nabla^2 G - G\nabla^2\Psi)\,d\tau' = \int_S (\Psi\,\vec{\nabla}G - G\,\vec{\nabla}\Psi)\,d\vec{a}'$$

$$\Psi(\vec{r}) = \varphi(\vec{r})$$
$$\nabla^2\Psi = \nabla^2\varphi = -\rho/\epsilon_0$$

$$\Psi\,\vec{\nabla}G - G\,\vec{\nabla}\Psi = 0$$

$$\nabla^2\varphi(R) = -\rho/\epsilon_0$$
$$\nabla^2 G(\vec{r},\vec{r}') = -\delta(\vec{R})$$

$$-\int \varphi(\vec{r}')\cdot\delta(\vec{r}-\vec{r}')\,d\tau' = -\frac{1}{\epsilon_0}\int_{V'} G(\vec{r},\vec{r}')\,\rho(\vec{r}')\,d\tau'$$

$$\longrightarrow \varphi(\vec{r}) = \frac{1}{\epsilon_0}\int_{V'} G(\vec{r},\vec{r}')\,\rho(\vec{r}')\,d\tau$$

symmetric wrt $\vec{r}$ & $\vec{r}'$