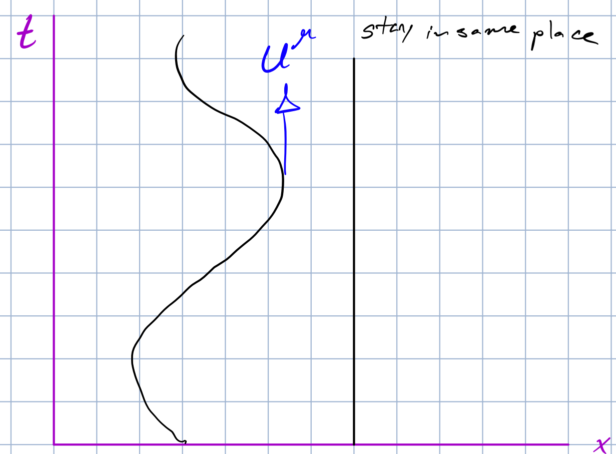
$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xleftrightarrow{GR} g_{\mu\nu}$$

$$\eta_{\alpha\gamma}\eta^{\gamma\beta} = \delta_\alpha^{\ \beta} = \delta^{\alpha\beta} = \mathbb{1}$$

$$(ds)^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$
$$= -(dx^0)^2 + \sum_i (dx^i)^2$$
$$= -(c\,dt)^2 + \sum_i (dx^i)^2$$

$$\|dx^\mu\|^2 = \eta_{\mu\nu}\, dx^\mu\, dx^\nu$$



u^μ – proper velocity

$$v^\mu = \frac{dx^\mu}{dt}$$

$$u^\mu = \frac{dx^\mu}{d\tau}$$ τ – proper time

$$dt = \gamma\, d\tau$$

time $\longleftrightarrow$ length of invariant interval

$$(d\tau)^2 = -\frac{1}{c^2}(ds)^2$$

max speed in 4 dim given by light

$$\left(\frac{ds}{d\tau}\right)^2 = -c^2$$

$$\left(\frac{c\,d\tau}{c\,dt}\right)^2 = \frac{c^2(dt)^2 - (dx^i)^2}{c^2(dt)^2}$$
$$= 1 - \left(\frac{dx^i}{c\,dt}\right)\cdot\left(\frac{dx^i}{c\,dt}\right)$$
$$= 1 - \frac{1}{c^2}\cdot \frac{dx^i}{dt}\circ\frac{dx^i}{dt}$$

$$dt = \gamma\, d\tau$$

$$u^\mu = \frac{1}{\gamma}\cdot\frac{dx^\mu}{dt} = \frac{dx^\mu}{d\tau}$$

Kinematics: $p^\mu = m \cdot u^\mu$ (momentum)

$$= (\gamma m c,\ \gamma m v,\ 0, 0)$$

Energy (relativistic)

$$E^{rel} = \gamma m c^2 = E_{rest} + E_{kin} = c p^0$$

Energy (non-rel)

$$\gamma = (1-\beta^2)^{-1/2} \simeq 1 + \tfrac{1}{2}\beta^2 + \tfrac{3}{8}\beta^4$$

$$E_{kin} = E^{rel} - E_{rest} = E_{rel} - mc^2$$

$$= mc^2(\gamma - 1) \simeq \tfrac{1}{2} m v^2 + \tfrac{3}{8}\frac{v^4}{c^2} m + \ldots$$

$$p^\mu p_\mu = m^2\, u^\mu u_\mu$$

$$u^\mu u_\mu = -(u^0)^2 + \sum (u^i)^2$$

$$= -\left(\gamma \frac{dx^0}{dt}\right)^2 + \sum \gamma^2 \left(\frac{dx^i}{dt}\right)^2$$

$$u^\mu u_\mu = -c^2$$

$$p^\mu p_\mu = -m^2c^2$$

$$-(p^0)^2 + (p^0)^2 = -m^2c^2$$

$$E^2 - \cancel{c^2 p^2} = m^2 c^4$$

@ rest

↳ $E = mc^2$