Poynting vector for dipole

$$\langle \vec{S} \rangle = \left(\frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c}\right)\left(\frac{\sin^2\theta}{r^2}\right)\hat{r}$$

$$\langle P \rangle = \left(\frac{\mu_0 p_0^2 \omega^4}{12\pi c}\right)$$

magnetic dipole instead of charges?
$p_0 \rightarrow m_0$

generalized radiation fields

$$\varphi(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}', t_r)}{R}\, d\tau'$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}', t_r)}{R}\, d\tau'$$

oscillating structure in current: $\vec{J}(\vec{r}',t') = \vec{J}(\vec{r}')\, e^{-i\omega t'}$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int \vec{J}(\vec{r}')\cdot \frac{e^{-i\omega(t-R/c)}}{R}\, d\tau'$$

$k = \omega/c$

$$= \frac{\mu_0}{4\pi}\left(e^{-i\omega t}\right)\cdot \int \vec{J}(\vec{r}')\cdot\left(\frac{e^{ikR}}{R}\right) d\tau'$$

$$\vec{A}(\vec{r},t) = \vec{A}(\vec{r})\cdot e^{-i\omega t}$$

$r \gg \lambda$
many coherent oscillations before measuring

$k\cdot r \gg 1$

$e^{ik|\vec{r}-\vec{r}'|}$

makes exponential large
hella oscillations
small wavelengths

$\vec{r} \gg \vec{r}'$ far away from source
dipole-esque

$$|\vec{R}| = |\vec{r}-\vec{r}'| = \left(r^2 - 2\vec{r}\cdot\vec{r}' + r'^2\right)^{1/2}$$

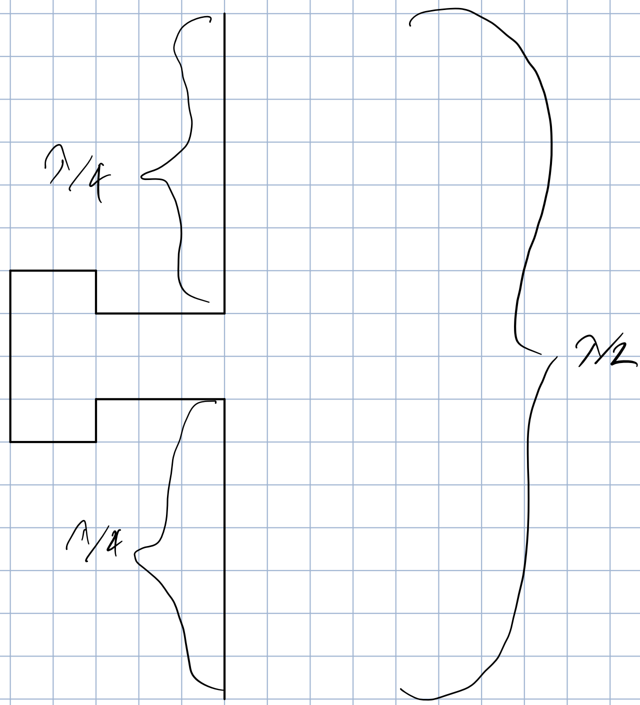
$$= r\left[1 - \frac{\vec{r}\cdot\vec{r}'}{r} + \frac{r'^2}{2r^2} - \frac{1}{8}\left(\frac{2\vec{r}\cdot\vec{r}'}{r}\right) \dots\right]^{1/2}$$

$$= r\left[1 - \frac{r'}{r}\cos\theta + \boxed{\frac{1}{2}\left(\frac{r'}{r}\right)^2 \sin^2\theta} + \dots\right]^{1/2}$$

$$e^{ik|\vec{r}-\vec{r}'|} \rightarrow e^{ik\vec{r}} \rightarrow \exp\left[\frac{kr}{2}\left(\frac{r'}{r}\right)^2 \sin^2\theta\right]$$

$\rightarrow \quad \frac{k}{2r}\cdot (r')^2 \ll 2\pi$

phase factors can come in, but have to be less than 2π



Fraunhofer Limit

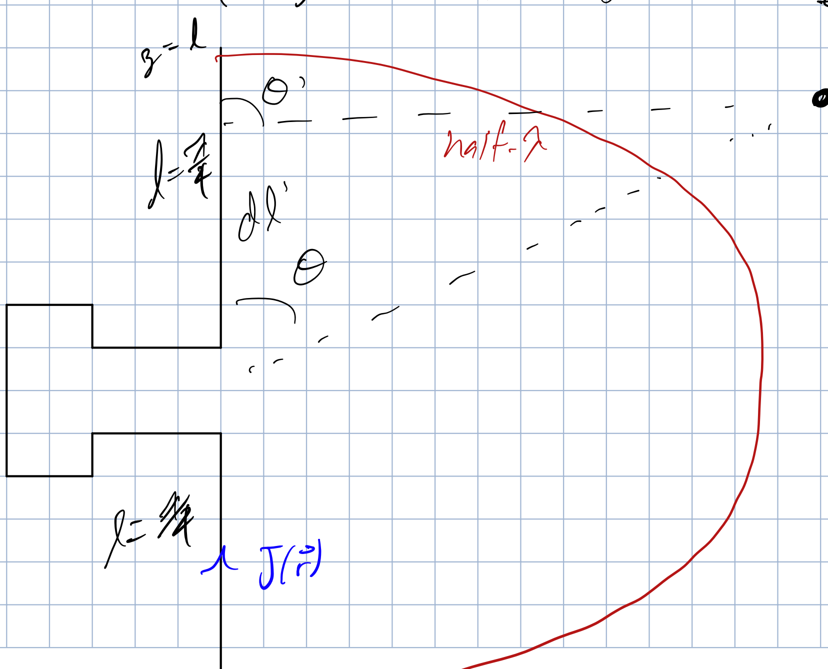
$$r \gg \frac{r'^2}{2\lambda} = \frac{d^2}{8\lambda}$$

d – dipole size

$\max(r') = d/2$

$$d \ll \sqrt{\lambda r} \ll r$$

$$\vec{A}(\vec{r}) \simeq \left(\frac{\mu_0}{4\pi}\right)\cdot\left(\frac{e^{ikr}}{r}\right)\cdot\int \vec{J}(\vec{r}')\,\frac{e^{-ik(\hat{r}\cdot\vec{r}')}}{e^{-ikr'\cos\theta}}\,d\tau'$$



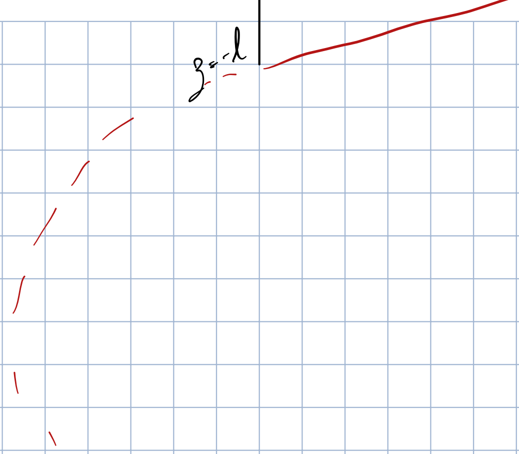
$I(t) = I_0 \cos(\omega t)$

$k = 2\pi/\lambda = \pi/2l$

$\lambda = 4l = 2d$

$I(z) = I_0 \cos\left(\frac{z}{l}\cdot\frac{\pi}{2}\right)$

each dl contributes different phase factors


z = -l