back @ it

mess w/ $\vec{B}$, make $\vec{E}$
mess w/ $\vec{E}$, make $\vec{B}$

Maxwell's change to Ampere: $\vec{\nabla}\times\vec{B} = \epsilon_0\mu_0 \partial_t \vec{E} + \mu_0 \vec{J}$

"real" – can move energy

Ch 2 of Griffiths: $U_E = \frac{\epsilon_0}{2}|\vec{E}|^2$

$U_B = \frac{1}{2\mu_0}|\vec{B}|^2$

remember order of magnitude: $U \propto E^2$
(energy density)

$U_{tot} = U_E + U_B = \frac{1}{2\mu_0}|\vec{B}|^2 + \frac{\epsilon_0}{2}|\vec{E}|^2$

$|\vec{E}|^2 = \vec{E}\cdot\vec{E}$

$\partial_t U_{tot} = \epsilon_0 \vec{E}\,\partial_t \vec{E} + \frac{1}{\mu_0}\vec{B}\,\partial_t \vec{B}$

$\vec{\nabla}(\vec{E}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{E}) - \vec{E}\cdot(\vec{\nabla}\times\vec{B})$

$\partial_t u_{tot} = -\frac{1}{\mu_0}[\vec{\nabla}\cdot(\vec{E}\times\vec{B})] - (\vec{E}\cdot\vec{J})$ energy density

$\frac{\partial U_{tot}}{\partial t} = -\int_V d\tau(\vec{E}\cdot\vec{J}) - \frac{1}{\mu_0}\oint_S (\vec{E}\times\vec{B})\,d\vec{a}$

energy / time

work done on charges & currents

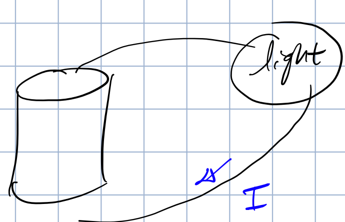
time rate of change of ~~total~~ energy

work done on charges & currents inside volume

have $\vec{E}$ & $\vec{B}$ field lines crossing surface
→ flow of energy / time!

flow of field thru surface of volume

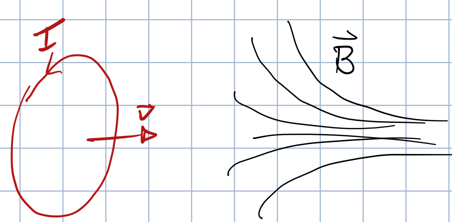
Electromotive Force EMF



need $\vec{E}$ to drive $\vec{I}$

$\vec{F}_E = q\vec{E}$

$\frac{\partial W}{\partial q} = \int \frac{\vec{F}}{q}\,d\vec{l} = \int \vec{E}\cdot d\vec{l} = -V$



need to do work to achieve current

induced current!
Φ_B increases

conducting loop from ∞

$$\mathcal{E}_{ind.} = -\frac{d\Phi_B}{dt} \qquad \Phi_B = \oint_S \vec{B}\cdot d\vec{a}$$

energy in $\vec{E}$ & $\vec{B}$:

ΔU = energy gained/lost $\qquad$ (u = energy density)

$$= -(\Delta q)(\Delta V)$$

$$\frac{dU}{dt} = \frac{dq}{dt}\cdot\left(-\vec{\nabla}V\cdot d\vec{l}\right) = \left(\vec{J}\cdot d\vec{a}\right)\left(-\vec{\nabla}\cdot d\vec{l}\right)$$

total energy: $\quad \frac{dU}{dt} = \left(-\vec{\nabla}V\cdot\vec{J}\right)\,d\tau$

energy density: $\quad \frac{du}{dt} = -\vec{\nabla}V\cdot\vec{J}$

express $\vec{\nabla}V$ in terms of $\vec{E}$ & $\vec{B}$

$$\left.\begin{array}{l}\vec{B} = \vec{\nabla}\times\vec{A}\\ \vec{\nabla}\times\vec{E} = -\partial_t\vec{B}\end{array}\right\} \quad \vec{\nabla}\times\vec{E} = -\partial_t\,\vec{\nabla}\times\vec{A}$$

$$\longrightarrow \vec{\nabla}\times\left(\vec{E}+\partial_t\vec{A}\right) = 0$$

$$\longrightarrow \vec{E}+\partial_t\vec{A} = -\vec{\nabla}V$$

$$\frac{du}{dt} = \left(\vec{E}+\partial_t\vec{A}\right)\cdot\vec{J} = \underbrace{\vec{E}\cdot\vec{J}}_{u_E} + \underbrace{\partial_t\vec{A}\cdot\vec{J}}_{u_B}$$

electric power density $\qquad$ magnetic power density

electric power density $\qquad \vec{J} = \sigma_{cond}\vec{E}$

$$P_E = \frac{du_E}{dt} = \vec{E}\cdot\vec{J} = \sigma_{cond}\,|\vec{E}|^2 = \underbrace{\frac{1}{\sigma_{cond}}}_{\propto\,\text{Resistance}}\cdot|\vec{J}|^2 \propto I^2$$

V / I R: $V = IR$ $\qquad$ P / I V: $P = IV = I^2R$

magnetic power density

$$P_M = \frac{du_M}{dt} = \partial_t\vec{A}\cdot\vec{J}_{tot} \overset{\text{in notes}}{=} \frac{1}{2}\,\partial_t\left(\vec{A}\cdot\vec{J}\right) = \frac{1}{2}\,\partial_t\left[\vec{A}\cdot\left(\frac{1}{\mu_0}\vec{\nabla}\times\vec{B}\right)\right]$$

$$= \frac{1}{2\mu_0}\,\partial_t\left[\vec{B}\cdot\left(\vec{\nabla}\times\vec{A}\right) - \vec{\nabla}\cdot\left(\vec{A}\times\vec{B}\right)\right]$$

$$\underbrace{\frac{dU_M}{dt}}_{\text{stored energy}} = \int_V \frac{du_M}{dt}\,d\tau = \frac{1}{2\mu_0}\,\partial_t\left[\underbrace{-\oint_S\left(\vec{A}\times\vec{B}\right)d\vec{a}}_{\text{source of field}} + \underbrace{\int_V \vec{B}\cdot\underbrace{\left(\vec{\nabla}\times\vec{A}\right)}_{=\vec{B}}\,d\tau}_{\text{field itself}}\right]$$

$$\frac{\partial U_{tot}}{\partial t} = -\int (\vec{E} \cdot \vec{J})\, d\tau - \frac{1}{\mu_0} \oint_S (\vec{E} \times \vec{B})\, d\vec{a}$$

time rate of change of total energy in fields

work done on charges & currents inside volume

flow of field energy thru surface of volume

$$\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$$

energy per unit time per unit area transported by the fields