Poisson eqn: $\nabla^2 \psi = -\frac{\rho}{\epsilon_0}$

Helmholtz: $(\nabla^2 - k^2)\psi = 0$

Wave Eqn: $(\nabla^2 - \frac{1}{v^2}\partial_t^2)\psi = 0$

Green's Function:

Poisson $\nabla^2 G = -\delta(\vec{r}_1 - \vec{r}_2) \rightarrow G = \frac{1}{4\pi(\vec{r}_1 - \vec{r}_2)} \rightarrow \psi = \frac{1}{\epsilon_0} \cdot \int G \cdot \rho \, d\tau$

Maxwell's Eqn

in vaccum: $\rho = 0$

$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\nabla^2 \vec{E}$

$\vec{\nabla} \times (\partial_t \vec{B}) = -\nabla^2 \vec{E}$

$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \partial_t^2 \vec{E}$

$\nabla^2 \vec{B} = \mu_0 \epsilon_0 \partial_t^2 \vec{B}$

$[\partial x] = L^{-1} \rightarrow [\nabla^2] = L^{-2}$

$[\nabla^2 E] = [E] \cdot L^{-2}$ $\quad$ $[\partial_t^2 E] = [E] \cdot T^{-2}$

$[\nabla^2 E] = [\mu_0 \epsilon_0] [\partial_t^2 E]$

$[E] L^{-2} = [\mu_0 \epsilon_0] [E] T^{-2}$

inverse speed

$\rightarrow [\mu_0 \epsilon_0] = L^{-2} T^2 = [v]^2$

$\mu_0 \epsilon_0 \equiv \frac{1}{c^2}$

$\nabla^2 \vec{E} = \frac{1}{c^2} \partial_t^2 \vec{E}$ $\quad$ $\nabla^2 \vec{B} = \frac{1}{c^2} \partial_t^2 \vec{B}$

z - 1 of 3 spatial dims.

could be complex

$\tilde{\vec{E}}(z,t) = \tilde{\vec{E}}_0 \, e^{i(kz - \omega t)}$

δ phase shift absorbed into $\tilde{\vec{E}}_0$ kinda like some initial condition

$\tilde{\vec{B}}(z,t) = \tilde{\vec{B}}_0 \, e^{i(kz - \omega t)}$

make sure they obey Maxwell's eqn's $\rightarrow$ establish BC.

$\vec{\nabla} \cdot \tilde{\vec{E}} = \frac{\rho}{\epsilon_0} = 0 \quad (\rho = 0)$

$\vec{\nabla} \cdot \tilde{\vec{B}} = 0$

$\vec{\nabla} \cdot \tilde{\vec{E}} = \sum_i \partial x_i (\tilde{E}_i) = 0$

$\cancel{\frac{\partial}{\partial x}(\tilde{E}_0)_x} + \cancel{\frac{\partial}{\partial y}(\tilde{E}_0)_y} + \frac{\partial}{\partial z}(\tilde{E})_z = 0$

0

$\partial_z(\tilde{E}_0 e^{i(kz-\omega t)}) = ik(\tilde{E}_0)_z = 0$

z component (if $\tilde{\vec{E}} = \tilde{\vec{E}}(z)$, not f^n of x, y)

amplitude of $\vec{E}$ in direction of propagation is zero

can point in x & y, but is only f^n of z. just showed $\tilde{\vec{E}}$ doesn't point in $\hat{z}$.

same w/ $\tilde{\vec{B}}$! $\quad (\tilde{B}_0)_z = 0$

$\vec{\nabla} \times \vec{E} = -\partial_t \vec{B}$

drop ∂x ∂y terms b/c it's zero

$\longrightarrow \quad -\partial_z(\tilde{E}_0)_y \hat{x} + \partial_z(\tilde{E}_0)_x \hat{y} = -\partial_t \vec{B}$

$\hat{x}$ comp.

$-\partial_z(\tilde{E}_0)_y \hat{x} = -\partial_t(\vec{B})_x \hat{x}$

$\longrightarrow \quad -k(\tilde{E}_0)_y = \omega(\tilde{B}_0)_x$

relation of 2 diff components!

$\hat{y}$ comp

$\partial_z(\tilde{E}_0)_x \hat{x} = -\partial_t(\vec{B})_y \hat{y}$

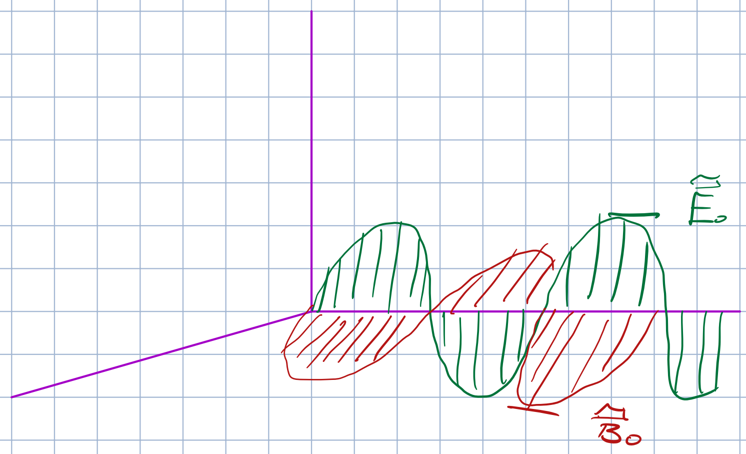
$\longrightarrow \quad k(\tilde{E}_0)_x = \omega(\tilde{B}_0)_y$

conclusion: $\tilde{\vec{B}}_0 = \frac{1}{c} \cdot (\hat{z} \times \tilde{\vec{E}}_0)$

$\tilde{\vec{E}} = \tilde{E}_0 e^{i(kz-\omega t)} \hat{x}$

$\tilde{\vec{B}} = \tilde{B}_0 e^{i(kz-\omega t)} \hat{y}$

always in phase!


$\vec{E}_0$
$\vec{B}_0$