MW stress tensor

$$\frac{d\vec{p}}{dt} = \vec{F} = \oint_S \overleftrightarrow{T} \cdot d\vec{a} - \epsilon_0 \mu_0 \frac{d}{dt} \int_V \vec{S}\, d\tau$$

relates incoming fields & force

time rate of change of momentum stored in fields

$\rightarrow \equiv \vec{p}^{\,fields}$

flow of momentum thru surfaces

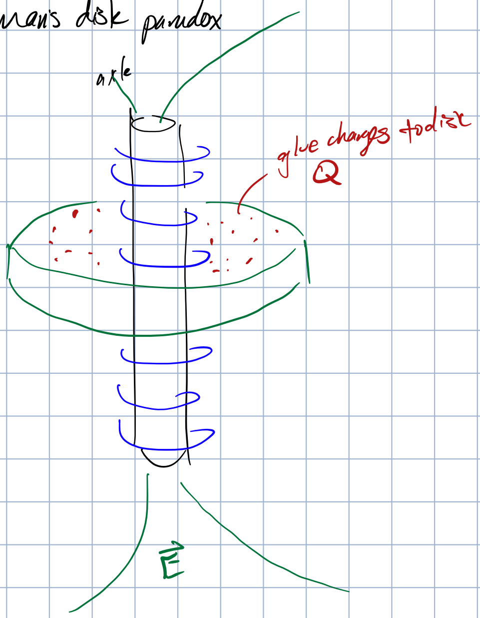
EM field only interacts w/something having electric charge

$$\frac{d\vec{p}^{\,charges}}{dt} = -\frac{d\vec{p}^{\,fields}}{dt} + \oint_S \overleftrightarrow{T} \cdot d\vec{a}$$

$$\frac{d}{dt}\left[\vec{p}^{\,charges} + \vec{p}^{\,fields}\right] = \oint \overleftrightarrow{T} \cdot d\vec{a}$$

$$\frac{d}{dt}\left[\vec{\mathcal{P}}^{\,charges} + \vec{\mathcal{P}}^{\,fields}\right] = \vec{\nabla} \cdot \overleftrightarrow{T}$$

Feynman's disk paradox



wrap in solenoid $\rightarrow \vec{B}$

$\frac{d\vec{B}}{dt} \neq 0$

$\frac{dB}{dt}$ in $\vec{S}$ is nonzero

$\vec{\nabla} \times \vec{B} \rightarrow \vec{E}!$

$\frac{d\vec{B}}{dt}$ in $\vec{E}$

spin! where does angular momentum come from?

fields already had angular momentum

e^- & γ have 2 angular momenta

Griffiths 8.A

$\vec{L} = \vec{r} \times \vec{p}$ $\qquad (\vec{p} = m\vec{v})$

$\vec{\mathcal{P}}^{\,fields} = \mu_0 \epsilon_0 \vec{S}$

for solenoid

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) = \left(\frac{Q n I}{2\pi \epsilon_0 l}\right) \frac{1}{r} (\vec{r} \times \hat{z})$$

l = length of solenoid

$$\vec{l} = \vec{r} \times \vec{\mathcal{P}} \rightarrow \vec{L} = \int_V \vec{l}\, d\tau$$

$\vec{l}$ → angular momentum density

EM Wave Eqn

1D : $\partial_x^2 f = \frac{1}{v^2}\partial_t^2 f$

$L^{-2} = T^2 L^{-2} \cdot T^{-2}$

3D : $\nabla^2 f = \frac{1}{v^2}\partial_t^2 f$

$v \rightarrow$ velocity of propagation

$f(z,t) = g(z-vt) + h(z+vt)$

$f(z,t) = A\cos(kz - \omega t + \delta)$

(k: wave #, ω: frequency, δ: phase)

complex notation: $f(z,t) = \mathrm{Re}\left[A\exp(i(kz-\omega t+\delta))\right]$

$\tilde{f}(z,t) = \tilde{A}\left[\text{" "}\right] = \int_{-\infty}^{\infty} \tilde{A}(k)\, e^{i(kz-\omega t)}\, dk$