Lenz's Law: induced currents oppose flux change

inductance: $\Phi_E = M \cdot I \qquad \Phi_B = L \cdot I$

energy in $\vec{E}$ & $\vec{B}$: $\quad \frac{du}{dt} = -\vec{\nabla} V \cdot \vec{J} \;;\; U_M = -\frac{1}{2\mu_0} \oint_S (\vec{A} \times \vec{B}) d\vec{a} + \frac{1}{2\mu_0} \int_V |\vec{B}|^2 d\tau$

Poynting: $\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$ energy per time per area in an EM field

$\partial_t U_{fields} = -\int (\vec{E} \cdot \vec{J}) d\tau - \frac{1}{\mu_0} \oint_S (\vec{E} \times \vec{B}) \cdot d\vec{a}$

$= -\partial_t U_{charges} - \oint_S \vec{S} \cdot d\vec{a}$

Topical Discussion
Tuesday 5:30 pm
Wednesday 3:30pm

$\frac{\partial}{\partial t} \left(\int_V u_{fields} d\tau + \int_V u_{charges} d\tau \right) = -\oint \vec{S} \cdot d\vec{a}$

Problem Solving <PTC 307
Wed 5:30 pm
Thu 3:30 pm
Thu 4:30 pm

$\frac{\partial}{\partial t} (u_{fields} + u_{charges}) = -\vec{\nabla} \cdot \vec{S}$ Poynting's $Th^{\underline{m}}$

conservation of energy: $\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot \vec{J}$ continuity $eq^{\underline{n}}$

if fields carry energy, can they also carry momentum?

Maxwell Stress Tensor

$\vec{F} = \int_V (\vec{E} + (\vec{v} \times \vec{B})) \rho \, d\tau$

$\vec{f} = \rho \vec{E} + \vec{J} \times \vec{B}$ $\qquad \vec{f}$ - force per unit volume

$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \partial_t \vec{E}$

$\partial_t (\vec{E} \times \vec{B}) = (\partial_t \vec{E} \times \vec{B}) + (\vec{E} \times \partial_t \vec{B}) = (\partial_t \vec{E} \times \vec{B}) + (\vec{E} \times (-\vec{\nabla} \times \vec{E}))$

$\vec{f} = \epsilon_0 (\vec{\nabla} \cdot \vec{E}) \cdot \vec{E} + \left[\frac{1}{\mu_0} (\vec{\nabla} \times \vec{B}) - \epsilon_0 \partial_t \vec{E} \right] \underline{\times \vec{B}}$

$= \epsilon_0 (\vec{\nabla} \cdot \vec{E}) \vec{E} - \frac{1}{\mu_0} \vec{B} \times (\vec{\nabla} \times \vec{B}) - \epsilon_0 \partial_t (\vec{E} \times \vec{B}) - \epsilon_0 \vec{E} \times (\vec{\nabla} \times \vec{E})$

$\underline{\times \vec{B} \text{ distribute}}$

$\vec{E} \times (\vec{\nabla} \times \vec{E}) = \frac{1}{2} \vec{\nabla} (\vec{E}^2) - (\vec{E} \cdot \vec{\nabla}) \vec{E}$

$$\vec{f} = \epsilon_0 \left[(\vec{\nabla} \cdot \vec{E}) \vec{E} + (\vec{E} \cdot \vec{\nabla}) \vec{E} \right] + \frac{1}{\mu_0} \left[(\vec{\nabla} \cdot \vec{B}) \cdot \vec{B} + (\vec{B} \cdot \vec{\nabla}) \cdot \vec{B} \right] \quad \leftarrow \text{spatial}$$

$$- \frac{1}{2} \vec{\nabla} \underbrace{\left[\epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right]} - \epsilon_0 \partial_t (\vec{E} \times \vec{B}) \quad \leftarrow \text{time}$$

$\propto (\text{field})^2$

gradients of expressions
→ cross terms

flow of momentum density

$$\vec{f} = \vec{\nabla} \cdot \overleftrightarrow{T} - \epsilon_0 \mu_0 \partial_t \vec{S}$$

$$\vec{F} = \oint_S \overleftrightarrow{T} \cdot d\vec{a} - \epsilon_0 \mu_0 \partial_t \int \vec{S} \, d\tau$$

$$T_{ij} = \epsilon_0 \left[E_i E_j - \frac{1}{2} \delta_{ij} E^2 \right] + \frac{1}{\mu_0} \left[B_i B_j - \frac{1}{2} \delta_{ij} B^2 \right]$$