dckk

radiation & gauges
$\hookrightarrow$ energy $\to \infty$
gauges $\hookrightarrow$ choices

sources

$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$ $\qquad$ $\vec{\nabla}\times\vec{E} = -\partial_t \vec{B}$

$\vec{\nabla}\cdot\vec{B} = 0$ $\qquad$ $\vec{\nabla}\times\vec{B} = \mu\epsilon\,\partial_t \vec{E} + \mu \vec{J}$

wave eq^n's $\quad \nabla^2 \vec{E} = \frac{1}{v}\partial_t^2 \vec{E}$

$\nabla^2 \vec{B} = \frac{1}{v}\partial_t^2 \vec{B}$

① have 3 components for each $\vec{E}$ & $\vec{B}$ $\to$ 6 eq^{n}s

with $\vec{A}$ & φ : 3 components for $\vec{A}$ & 1 φ $\to$ 4 eq^n's

② $\vec{E}, \vec{B} \propto 1/r^2$ $\qquad$ $\vec{A}, \varphi \propto 1/r$

③ Sources are in wave eq^n

④ relativistic forms: $\quad A^\mu = (\varphi, \vec{A})$

$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

⑤ Gauge invariance is clearer $\qquad$ group theory w/ Lie Groups

Potentials $\vec{A}$, φ

$\vec{\nabla}\cdot\vec{B} = 0 \quad \longleftrightarrow \quad \vec{B} = \vec{\nabla}\times\vec{A}$

$\vec{\nabla}\times\vec{E} = 0 = -\partial_t(\vec{\nabla}\times\vec{A})$

$\vec{\nabla}\times\vec{E} + \partial_t(\vec{\nabla}\times\vec{A}) = \vec{\nabla}\times(\vec{E} + \partial_t\vec{A}) = 0$

$\vec{E} = -\vec{\nabla}\varphi - \partial_t\vec{A}$

$\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \mu_0\epsilon_0\,\partial_t(-\vec{\nabla}\varphi - \partial_t\vec{A}) + \mu_0\vec{J}$

$\vec{\nabla}\cdot(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A} = -\mu_0\epsilon_0\vec{\nabla}(\partial_t\varphi) - \mu_0\epsilon_0\,\partial_t^2\vec{A} + \mu_0\vec{J}$

$\vec{\nabla}\cdot(-\vec{\nabla}\varphi - \partial_t\vec{A}) = \rho/\epsilon_0$

$\nabla^2\varphi + \partial_t(\vec{\nabla}\cdot\vec{A}) = \rho/\epsilon_0$

What about: $\vec{A} \rightarrow \vec{A} + \vec{\nabla}\psi \qquad \varphi \rightarrow \varphi - \partial_t\psi$

$$\vec{E} = -\vec{\nabla}(\varphi - \partial_t\psi) - \partial_t(\vec{A} + \vec{\nabla}\psi)$$

$$= -\vec{\nabla}\varphi - \partial_t\vec{A} + \cancel{\vec{\nabla}(\partial_t\psi) - \vec{\nabla}(\partial_t\psi)}$$

$$\vec{B} = \vec{\nabla}\times(\vec{A} + \vec{\nabla}\psi) = \vec{\nabla}\times\vec{A} + \cancel{\vec{\nabla}\times(\vec{\nabla}\psi)}$$

choose Lorenz gauge

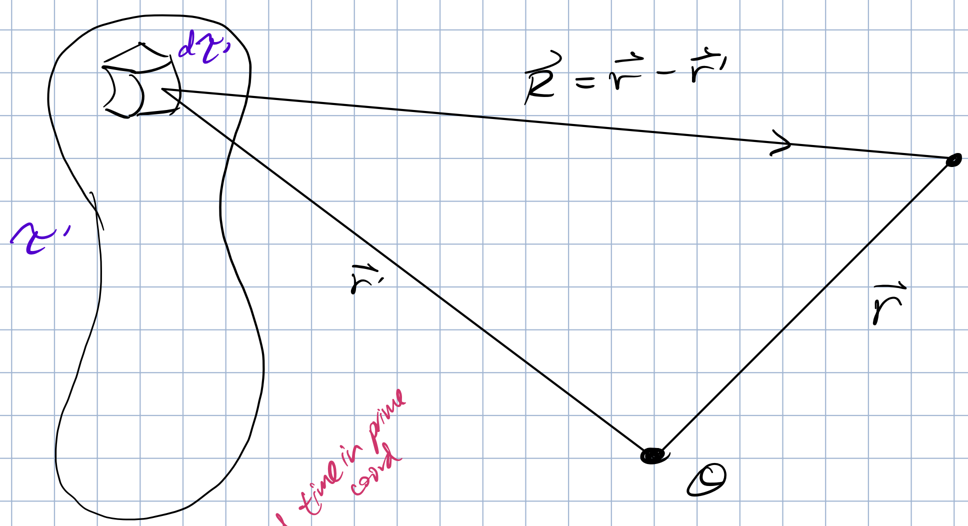
$$\vec{\nabla}\cdot\vec{A} = -\mu_0\epsilon_0\partial_t\varphi$$

$$\nabla^2\vec{A} - \frac{1}{c^2}\partial_t^2\vec{A} = -\mu_0\vec{J}$$

$$\nabla^2\varphi - \frac{1}{c^2}\partial_t^2\varphi = -\rho/\epsilon_0$$

$$\Box \equiv \frac{1}{c^2}\partial_t^2 - \nabla^2$$

$$\Box\vec{A} = \mu_0\vec{J} \quad \& \quad \Box\varphi = \rho/\epsilon_0$$



time in prime coord

$$t_{delay} = |\vec{R}|/c$$

$$t_{ret} = t' = t - t_{delay} = t - |\vec{R}|/c$$