# MWE in conductors!

$$\nabla\cdot\tilde{E} = \frac{\rho_f}{\epsilon}$$
$$\nabla\cdot\tilde{B} = 0$$
$$\nabla\times\tilde{E} = -\partial_t \tilde{B}$$
$$\nabla\times\tilde{B} = \epsilon\mu\,\partial_t\tilde{E} + \mu\tilde{J}_f \;\longrightarrow\; = \underbrace{\sigma_f}_{\text{conductivity}}\tilde{E}$$

$$\nabla\cdot\vec{J}_f = -\partial_t\rho_f$$
$$\nabla\cdot(\sigma\vec{E}) = -\partial_t\rho_f$$
$$\frac{\sigma}{\epsilon}\rho_f = -\partial_t\rho_f$$
$$\longrightarrow \rho_f(t) = \rho_f(t=0)\,e^{-(\sigma/\epsilon)t}$$

## Wave Eqn

$$\nabla^2\tilde{E} = \mu\epsilon\partial_t^2\tilde{E} + \mu\sigma\partial_t\tilde{E}$$
$$\nabla^2\tilde{B} = \mu\epsilon\partial_t^2\tilde{B} + \mu\sigma\partial_t\tilde{B}$$

$\rho_f = 0$

LHS: $\nabla^2\tilde{E} = -k^2\tilde{E}$

RHS: $\mu\epsilon\partial_t^2\tilde{E} = -\omega^2\mu\epsilon\tilde{E}$

$\mu\sigma\partial_t\tilde{E} = -i\mu\sigma\omega\tilde{E}$

$$\longrightarrow \quad -k^2 = -\omega^2\mu\epsilon - i\mu\sigma\omega$$

say $k$ is complex: $\tilde{k} = \pm(k \pm i\underbrace{\kappa}_{\text{kappa}})$

plug in & solve quadratic

$$k^2 = \frac{\omega^2}{2}\epsilon\mu\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2} + 1\right]$$
$$\kappa^2 = \frac{\omega^2}{2}\epsilon\mu\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2} - 1\right]$$

$\left[\frac{\sigma}{\epsilon}\right] \sim T^{-1}$

$Q^{-1} \equiv \left(\frac{\sigma}{\epsilon\omega}\right)$ "quality factor" dimensionless

relates displacement vs conductor current
$\sigma\partial_t E$ vs $\sigma E$

① $Q \gg 1 \longrightarrow$ insulator
conductivity small
frequency of waves is large

② $Q \ll 1 \longrightarrow$ conductor

$[\text{wave vector}] = L^{-1} = [k] = [\kappa]$

$$\frac{1}{\kappa^2} = d_{skin}^2 \;(\text{length}) = \left(\frac{2}{\omega^2\epsilon\mu}\right)\cdot\left[\sqrt{1+\frac{1}{Q^2}} - 1\right]^{-1}$$

$$d_{skin} = \left(\frac{2}{\epsilon\mu\omega^2}\right)^{1/2}\cdot\left[\sqrt{1+\frac{1}{Q}} - 1\right]^{-1/2}$$

① $Q \gg 1$: $\sigma \ll \omega\epsilon$

$$\kappa \simeq \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} \longrightarrow d_{skin} \simeq \frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$$

② $Q \ll 1$: $\sigma \gg \omega\epsilon$

$$\kappa \simeq \sqrt{\frac{\mu\sigma\omega}{2}} \longrightarrow d_{skin} \simeq \sqrt{\frac{2}{\sigma\mu\omega}}$$

$$\tilde{\vec{E}}(z,t) = \tilde{E}_0 \underbrace{e^{-\kappa z}} e^{i(kz-\omega t)} \hat{x}$$

$$\tilde{\vec{B}}(z,t) = \overset{???}{\frac{k^2}{\omega}} \tilde{E}_0 \underbrace{e^{-\kappa z}} e^{i(kz-\omega t)} \hat{y}$$

killing $\tilde{\vec{B}}$ & $\tilde{\vec{E}}$ going into material ($z$ grows)

acts as damping term

$$\vec{\nabla}\times\tilde{\vec{E}} = (+ik-\kappa)\tilde{\vec{E}}$$
$$-\partial_t \tilde{\vec{B}} = +i\omega\tilde{\vec{B}}$$

$$\longrightarrow \tilde{B}_0 = \frac{1}{\omega}|ik+\kappa|\tilde{E}_0$$

just a phase factor! E & B related by complex num

phase no longer fixed!

$$\tilde{\kappa} = K e^{i\phi} = K(\cos\phi + i\sin\phi)$$

$$K^2 = |\tilde{\kappa}|^2 = k^2 + \kappa^2 = \omega^2\left(\epsilon\mu\sqrt{1+(1/Q)^2}\right)$$

$$\varphi \equiv \tan^{-1}\left(\frac{\kappa}{k}\right) \qquad \tilde{k} = k + i\kappa$$