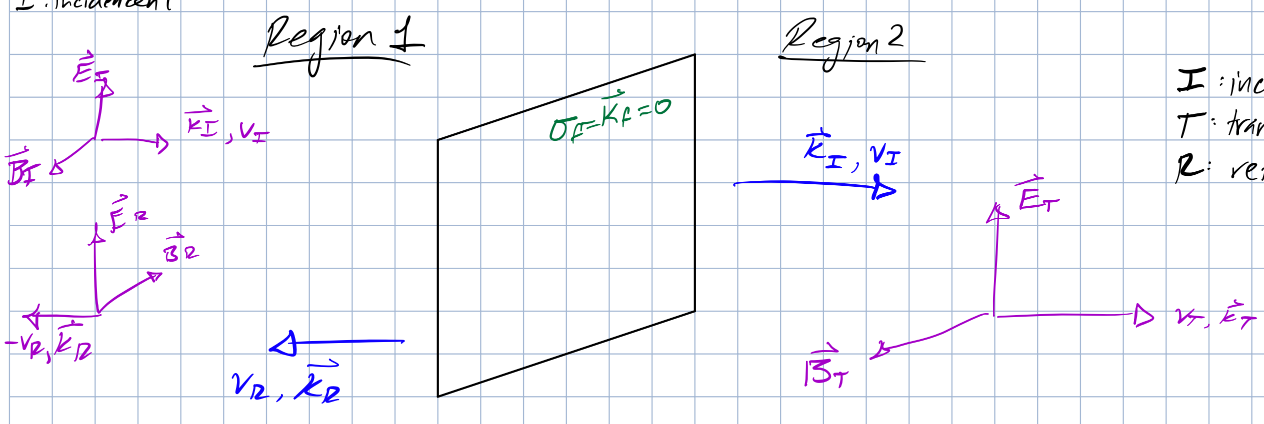
Wave eq^n's in LIH medium

$$\nabla^2 \vec{E} = \mu\epsilon\,\partial_t^2 \vec{B} \qquad v = \frac{c}{n}$$
$$\nabla^2 \vec{B} = \mu\epsilon\,\partial_t^2 \vec{E} \qquad n = \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}} \simeq \sqrt{\frac{\epsilon}{\epsilon_0}} \qquad \vec{D} = \epsilon\vec{E}$$

I: incidencent



incident $\measuredangle = 90°$

Fields in each region

$$v = \frac{1}{\sqrt{\mu\epsilon}}$$

incident:
$$\tilde{\vec{E}}_I(z,t) = \tilde{E}_{0,I}\, e^{i(k_I z - \omega_I t)}\,\hat{x}$$
$$\tilde{\vec{B}}_I(z,t) = \tilde{B}_{0,I}\, e^{i(k_I z - \omega_I t)}\,\hat{y} \qquad \tilde{B}_{0,I} = \frac{\tilde{E}_{0,I}}{v_I}$$

transmitted:
$$\tilde{\vec{E}}_T(z,t) = \tilde{E}_{0,T}\, e^{i(k_T z - \omega_T t)}\,\hat{x}$$
$$\tilde{\vec{B}}_T(z,t) = \tilde{B}_{0,T}\, e^{i(k_T z - \omega_T t)}\,\hat{y} \qquad \tilde{B}_{0,T} = \frac{\tilde{E}_{0,T}}{v_T}$$

reflected:
$$\tilde{\vec{E}}_R(z,t) = \tilde{E}_{0,R}\, e^{i(k_R z - \omega_R t)}\,\hat{x}$$
$$\tilde{\vec{B}}_R(z,t) = \tilde{B}_{0,R}\, e^{i(k_R z - \omega_R t)}\,\hat{y} \qquad \tilde{B}_{0,R} = \frac{\tilde{E}_{0,R}}{v_R}$$

BC #

gaussian surface

① $\oint_S \vec{D}\cdot d\vec{a} = Q_{f,encl.}$

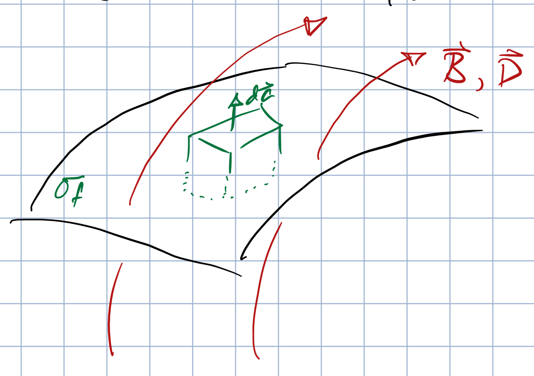
② $\oint \vec{B}\cdot d\vec{a} = 0$ (same in as out, $\nabla\cdot\vec{B} = 0$)

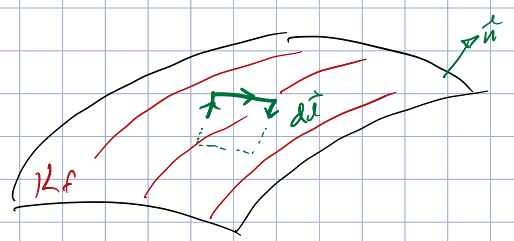
amperian loop

③ $\oint_\ell \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{a}$

④ $\oint_\ell \vec{H}\cdot d\vec{l} = I_{f,encl} + \frac{d}{dt}\int_S \vec{D}\cdot d\vec{a}$

now time dependence





① $\oint_S \vec{D}\cdot da \overset{LIH}{=} \oint_S \epsilon \vec{E}\cdot d\vec{a} = Q_{f,encl.}$ subscript 1 & 2 are regions

$\vec{D}_1 \cdot \vec{a}_1 + \vec{D}_2 \cdot \vec{a}_2 = \vec{D}_1 \vec{a} - D_2 \vec{a} = \sigma_f \cdot \vec{a}$

$D_1^{\perp} - D_2^{\perp} = \sigma_f$

$\epsilon_1 \vec{E}_1^{\perp} - \epsilon_2 \vec{E}_2^{\perp} = \sigma_f$ (nothing about parallel comp.)

② $\oint_S \vec{B}\cdot d\vec{a} = 0$

$\vec{B}_1^{\perp} - \vec{B}_2^{\perp} = 0$

③ $\oint_\ell \vec{E}\cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B}\cdot da$

→ 0 b/c can shrink loop area

$\vec{E}_1^{\parallel} - \vec{E}_2^{\parallel} = 0$ // comp. is continuous

④ $\int_\ell \vec{H}\cdot d\vec{l} = I_{f,encl} + \frac{d}{dt} \int_S \vec{D}\cdot da$

$\vec{H}_1^{\parallel} - \vec{H}_2^{\parallel} = I_{f,encl.} = \vec{K}_f \times \hat{n}$

$\frac{1}{\mu_1} \vec{B}_1^{\parallel} - \frac{1}{\mu_2} \vec{B}_2^{\parallel} = \vec{K}_f \times \hat{n}$

↳ norm to surface

$\hat{n}_1 = -\hat{z}$ $\hat{n}_2 = \hat{z}$

discontinuity @ $\vec{E}^{\perp}$ & $\vec{B}^{\parallel}$

continuity @ $\vec{E}^{\parallel}$ & $\vec{B}^{\perp}$

$\tilde{\vec{E}}_1 = \tilde{\vec{E}}_{0,I} + \tilde{\vec{E}}_{0,R}$ $\tilde{\vec{E}}_2 = \tilde{\vec{E}}_{0,T}$

$\tilde{E}_{0,I}\, e^{i(k_I z - \omega_I t)} + \tilde{E}_{0,R}\, e^{i(k_R z - \omega_R t)} = \tilde{E}_{0,T}\, e^{i(k_T z - \omega_T t)}$

using ③ → $\tilde{E}_1 = \tilde{E}_2$, can set boundary @ $z=0$

all $e^{i(k,z)}$ components die

→ $\omega_I = \omega_R = \omega_T \equiv \omega$