Radiation Pressure $\quad P^{rad} = c \langle \vec{P}^{fields} \rangle$

Polarization: preferred direction $\quad \hat{n} = \cos\theta\,\hat{x} + \sin\theta\,\hat{y}$

updated MWE

$$\left.\begin{aligned} \rho_b &= -\vec{\nabla}\cdot\vec{P} \\ \vec{D} &= \epsilon_0 \vec{E} + \vec{P} \\ \vec{\nabla}\cdot\vec{E} &= \frac{1}{\epsilon_0}\left(\rho_f - \vec{\nabla}\cdot\vec{P}\right) \end{aligned}\right\} \quad \vec{\nabla}\cdot\vec{D} = \rho_f$$

$$\left.\begin{aligned} \vec{J}_{b,M} &= \vec{\nabla}\times\vec{M} \\ \vec{H} &= \frac{1}{\mu_0}\vec{B} - \vec{M} \\ \vec{\nabla}\cdot\vec{J}_{b,P} &= -\partial_t(\rho_b) = \partial_t(\vec{\nabla}\cdot\vec{P}) \end{aligned}\right\} \quad \vec{J}_{b,P} = \partial_t \vec{P}$$

bound current due to time-dependent polarization

$$\vec{\nabla}\cdot\vec{D} = \rho_f$$

$$\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0 \partial_t \vec{E} + \mu_0\left(\vec{J}_f + \vec{J}_{b,M} + \vec{J}_{b,P}\right)$$

$$= \mu_0\epsilon_0 \partial_t \vec{E} + \mu_0 \vec{J}_f + \mu_0 \vec{\nabla}\times\vec{M} + \mu_0 \partial_t \vec{P} \qquad + \text{fields!}$$

$$\vec{\nabla}\times\left(\frac{1}{\mu_0}\vec{B} - \vec{M}\right) = \epsilon_0 \partial_t \vec{E} + \partial_t \vec{P} + \vec{J}_f$$

$$\vec{\nabla}\times\vec{H} = \partial_t\left(\epsilon_0\vec{E} + \vec{P}\right) + \vec{J}_f \qquad \text{updated}$$

$$\vec{\nabla}\times\vec{E} = -\partial_t \vec{B} \qquad \vec{\nabla}\cdot\vec{B} = 0 \qquad \text{still same}$$

Linear, Isotropic, Homogeneous (LIH) Materials

E&M's spherical cow

assume $\vec{D} = \epsilon_0\vec{E} + \vec{P}$ is true $\rightarrow$ $\vec{E}$ changes $\vec{P}$ linearly

$$\vec{D} = \epsilon \vec{E} \qquad \epsilon_0 \rightarrow \epsilon$$
$$\vec{H} = \mu \vec{B} \qquad \mu_0 \rightarrow \mu$$

$$\nabla^2 \tilde{\vec{E}} = \mu\epsilon\, \partial_t^2 \tilde{\vec{E}}$$

$$\nabla^2 \tilde{\vec{B}} = \mu\epsilon\, \partial_t^2 \tilde{\vec{B}}$$

$$[\ \ ] = \left[\frac{1}{v^2}\right]$$

$$\hookrightarrow v = \frac{1}{\sqrt{\mu\epsilon}}$$

$v^2 = \frac{1}{\mu \epsilon} \equiv \frac{c^2}{n^2} \rightarrow n = \sqrt{\frac{\epsilon \mu}{\epsilon_0 \mu_0}}$ most materials: $\mu \approx \mu_0$

$n \simeq \sqrt{\frac{\epsilon}{\epsilon_0}}$

$k = \frac{\omega}{v}$

$\tilde{B}(z,t) = \sqrt{\mu \epsilon}\, \tilde{E}_0\, e^{i(kz - \omega t)}\, \hat{y}$

$\frac{1}{v} = \frac{n}{c}$

assumptions

① material

② $\mu \approx \mu_0$

#3

$\tilde{\vec{E}}_1 = \tilde{E}_{0,1} \, e^{i(kz-\omega t)} \, \hat{x}$ $\qquad \hat{k} = \hat{z} \qquad \hat{n}_1 = \hat{x}$

$\tilde{\vec{E}}_2 = \tilde{E}_{0,2} \, e^{i(kz-\omega t+\delta)} \, \hat{y}$ $\qquad$ " " $\qquad \hat{n}_2 = \hat{y}$

$\tilde{\vec{E}}_3 = \tilde{E}_{0,3} \, e^{i(kz-\omega t+\delta)} \, \hat{x}$ $\qquad$ " " $\qquad \hat{n}_3 = \hat{x}$

$\tilde{\vec{E}}_4 = \tilde{E}_{0,4} \, e^{i(kz-\omega t)} \, \hat{x}$ $\qquad$ " " $\qquad \hat{n}_4 = \hat{x}$

a) #1, #2 $\qquad$ b) #1, #3 $\qquad$ c) #1, #4

$\frac{(E_{01}^2 + E_{02}^2)\hat{z}}{c} = \langle S \rangle$ $\qquad$ cos $\qquad$ cos

$$\vec{B} = \frac{1}{\mu_0} \vec{k} \times \vec{E} = \frac{\tilde{E}_{01}}{c} e^{i(kz-\omega t)} (\hat{k}_1 \times \hat{n}_1)$$

$$\langle S \rangle = \frac{1}{c} \, Re[\tilde{\vec{E}} \times \tilde{\vec{B}}]$$

$$\vec{B}_{12} = \vec{B}_{01} + \vec{B}_{02} = \frac{\tilde{E}_{01}}{c}(\hat{z} \times \hat{x}) + \frac{\tilde{E}_{02}}{c}(\hat{k} \times \hat{y}) = \underset{(\hat{y})}{} \qquad \underset{(-\hat{x})}{}$$

1 & 3

$$\rightarrow S = \frac{1}{\mu_0} Re[\tilde{\vec{E}}] \times Re[\tilde{\vec{B}}]$$

tuning phase $\rightarrow$ max/min

$$= \frac{1}{\mu_0}\left(E_{01} \cos(kz-\omega t)\,\hat{x} + E_{03} \cos(kz-\omega t+\delta)\,\hat{x} \right)$$
$$\times \frac{1}{c}\left(E_{01} \cos(kz-\omega t)(\hat{k} \times \hat{x}) + E_{03} \cos(kz-\omega t+\delta)(\hat{k} \times \hat{x}) \right)$$

$$\cos(kz-\omega t+\delta) = \cos(kz-\omega t)\cos(\delta) - \sin(kz-\omega t)\sin(\delta) \qquad (\hat{k} \times \hat{x}) = \hat{z} \times \hat{x} = \hat{y}$$

$$S = \frac{1}{c\mu_0}\left[E_{01} \cos(kz-\omega t)\,\hat{x} + E_{03} \cos(kz-\omega t)\cos(\delta)\,\hat{x} - E_{03} \sin(kz-\omega t)\sin(\delta)\,\hat{x} \right]$$
$$\times \left[E_{01} \cos(kz-\omega t)\,\hat{y} + E_{03} \cos(kz-\omega t)\cos(\delta)\,\hat{y} - E_{03} \sin(kz-\omega t)\sin(\delta)\,\hat{y} \right]$$

$$= \frac{1}{c\mu_0}\Big[E_{01}^2 \cos^2(kz-\omega t)(\hat{x} \times \hat{y}) + E_{03}E_{01} \cos^2(kz-\omega t)\cos(\delta)(\hat{x} \times \hat{y}) - E_{03}E_{01} \sin(kz-\omega t)\sin(\delta)\cos(kz-\omega t)(\hat{x} \times \hat{y})$$
$$+ E_{01}E_{03} \cos^2(kz-\omega t)\cos(\delta)(\hat{x} \times \hat{y}) + E_{03}^2 \cos(kz-\omega t)\cos(\delta)(\hat{x} \times \hat{y}) - E_{03}^2 \sin(kz-\omega t)\sin(\delta)\cos(kz-\omega t)\cos(\delta)(\hat{x} \times \hat{y})$$
$$- E_{01}E_{03} \cos(kz-\omega t)\sin(kz-\omega t)\sin(\delta)(\hat{x} \times \hat{y}) - E_{03}^2 \sin(kz-\omega t)\sin(\delta)\cos(kz-\omega t)\cos(\delta)(\hat{x} \times \hat{y}) + E_{03}^2 \sin(kz-\omega t)\sin(\delta)(\hat{x} \times \hat{y})\Big]$$

$$= \frac{1}{c\mu_0}\Big($$

$$\frac{1}{\mu_0}\left(\tilde{\vec{E}}_{01} + \tilde{\vec{E}}_{03}\right) \times \left(\tilde{\vec{B}}_{01} + \tilde{\vec{B}}_{03}\right)$$

$$\frac{1}{\mu_0 \cdot c}\, Re\left[\tilde{\vec{E}}_{01} + \tilde{\vec{E}}_{03}\right] \times Re\left[\vec{k} \times \tilde{\vec{E}}_{01} + \vec{k} \times \tilde{\vec{E}}_{03}\right]$$

$$\frac{1}{\mu_0 c} \cdot \left(E_{01}\cos(kz-\omega t) + E_{03}\cos(kz-\omega t+\delta)\right)\hat{x} \times \left(\hat{z} \times E_{01}\cos(kz-\omega t)\,\hat{x} + \hat{z} \times E_{03}\cos(kz-\omega t+\delta)\,\hat{x}\right)$$

$\hat{z} \times \hat{x} = \hat{y}$ $\qquad$ $\hat{z} \times \hat{x} = \hat{y}$

$$\frac{1}{\mu_0 c} \cdot \left(E_{01}\cos(kz-\omega t) + E_{03}\cos(kz-\omega t+\delta)\right)\hat{x} \times \left(E_{01}\cos(kz-\omega t) + E_{03}\cos(kz-\omega t+\delta)\right)\hat{y}$$

$$\frac{1}{\mu_0 c}\left(E_{01}\cos(kz-\omega t) + E_{03}\cos(kz-\omega t+\delta)\right)^2 \left(\hat{x} \times \hat{y}\right)$$

$\hat{x} \times \hat{y} = \hat{z} = \hat{z}$

$$\frac{1}{\mu_0 c} \cdot \left(E_{01}^2\cos^2(kz-\omega t) + E_{03}^2\cos^2(kz-\omega t+\delta) + E_{01}E_{03}\cos(kz-\omega t)\cos(kz-\omega t+\delta)\right)\hat{z}$$

#4

$$u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$

$$\vec{E} = E_0 \cos(\hat{k}\hat{x} - \omega t)\,\hat{n}$$
$$\vec{B} = \frac{E_0}{c} \cos(\hat{k}\hat{x} - \omega t)\,(\hat{k}\times\hat{n})$$

$$u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0 c^2} E_0^2\right) \cos^2(\hat{k}\hat{x} - \omega t)$$

$$= \epsilon_0 E_0^2 \cos^2(\hat{k}\hat{x} - \omega t)$$

$$\langle u \rangle = \frac{\epsilon_0 E_0^2}{2}$$

$$S = \frac{1}{\mu_0} \vec{E}\times\vec{B}$$

$$= \frac{E_0^2}{\mu_0 c} \cos^2(\hat{k}\hat{x} - \omega t)\cdot\hat{k}$$

$$\langle S \rangle = c\cdot\frac{\epsilon_0}{2} E_0^2\,\hat{k} = c u \hat{k}$$

$$\vec{P} = \frac{1}{c^2}\vec{S} \qquad \langle \vec{P} \rangle = \frac{u}{c}\hat{k}$$

$$153\ \frac{mW}{cm^2} \qquad \frac{\text{Energy}}{\text{area}\cdot\text{time}} = \langle S \rangle = c\,\frac{\epsilon_0}{2} E_0^2$$

$$E_0 = \frac{2}{\epsilon_0 \cdot c}\cdot 153\ \frac{mW}{cm^2}$$

$$\text{Pressure} = \frac{\text{Force}}{\text{area}}$$

$$= \left(\frac{\Delta P}{\Delta t}\right)\frac{1}{A} = \left(\frac{\frac{u}{c}\cdot\Delta t\cdot A}{\Delta t}\right)\frac{1}{A} = \frac{u}{c}$$

$$\text{Total Power} = (I)(4\pi R^2)$$

$$\text{Bright} = I = \langle S \rangle$$

$$I_b = I_s$$

$$\frac{P_?}{A_b} = \frac{P_3}{A_s}$$

#5

$$\vec{E} = \sum_k \vec{E}_k$$

$$\langle u \rangle = \sum_k \langle u_k \rangle$$

$$\left\langle \left(\sum_k \vec{E}_k\right)\cdot\left(\sum_k \vec{E}_k\right) \right\rangle \longrightarrow E_1^2 + E_1 E_2 + \ldots$$

why do cross terms vanish w/ average?

$$u = \frac{1}{2}\epsilon_0 |\vec{E}|^2$$

$$= \frac{1}{2}\epsilon_0 \sum_{k,k'} E_{0,k}\cos(kz-\omega t+\delta_k)\, E_{0,k'}\cos(k'z-\omega t+\delta_{k'})$$

$$\frac{1}{T}\int_0^T \cos(\omega_k t)\cos(\omega_{k'} t)\,dt = \delta_{k'k}\cdot\frac{1}{2}$$

$\omega_i = \frac{2\pi t}{T_i}$ $\quad$ T is a common multiple of T_k $T_{k'}$

T is LCM of T_k $T_{k'}$

$\cos(kz+\delta)\cos(\omega t) + \sin(kz+\delta)\sin(\omega t)$

cross terms $\rightarrow 0$ $\quad$ b/c $\int \sin \cos = 0$

$\cos\cos = \delta_{kk'}$

3

can do $\tilde{E}^*_{0,2} = \tilde{E}_{0,2}e^{i\delta}$

$$\tilde{\vec{E}} = \tilde{E}_{0,1}\, e^{i(kz-\omega t)}\hat{x} + \tilde{E}_{0,2}\, e^{i(kz-\omega t+\delta)}\hat{y}$$

$$\vec{B} = \frac{1}{c}\hat{k}\times\vec{E} = \frac{1}{c}\left(\tilde{E}_{0,1}\, e^{i(kz-\omega t)}\hat{y} + \tilde{E}_{02}\, e^{i(kz-\omega t+\delta)}(-\hat{x})\right)$$

$$\vec{S} = \frac{1}{\mu_0}\mathrm{Re}[\tilde{\vec{E}}]\times \mathrm{Re}[\tilde{\vec{B}}]$$

$$\langle \vec{S}\rangle = \frac{1}{2\mu_0}\mathrm{Re}(\tilde{\vec{E}}\times\tilde{\vec{B}}^*)$$ $\hat{k}$ in same dir

add, subtr, or nada

$\delta \rightarrow$ constr or dest interference