energy density

$$u = \frac{1}{2}\left[\epsilon_0 |\tilde{\vec{E}}|^2 + \frac{1}{\mu_0}|\tilde{\vec{B}}|^2\right] = \frac{\epsilon_0}{2}\left[|\tilde{\vec{E}}|^2 + \underbrace{\frac{1}{\mu_0\epsilon_0}}_{= c^2}|\tilde{\vec{B}}|^2\right]$$

$$\tilde{\vec{E}} \perp \tilde{\vec{B}} \longrightarrow u = \epsilon_0 |\tilde{\vec{E}}|^2 = \frac{\epsilon_0}{c^2}|\tilde{\vec{B}}|^2$$ (can interchange $\tilde{\vec{E}}$ & $\tilde{\vec{B}}$)

$$\mathrm{Re}[\tilde{\vec{E}}] = E_0 \cos(\vec{k}\cdot\vec{r} - \omega t)\,\hat{n}$$

$$\mathrm{Re}[\tilde{\vec{B}}] = \frac{1}{c} E_0 \cos(\vec{k}\cdot\vec{r} - \omega t)\,(\hat{k}\times\hat{n})$$

$\vec{k} = \hat{z}$ (direction of propagation) (set for example)

$$u = \epsilon_0 E_0^2 \cos^2(kz - \omega t)$$

$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$ → NOT complex

$$\vec{S} = \frac{1}{\mu_0}\left(\mathrm{Re}[\tilde{\vec{E}}]\times\mathrm{Re}[\tilde{\vec{B}}]\right) = c\cdot\epsilon_0 E_0^2 \cos(kz - \omega t)\cdot\hat{z}$$

energy flux density traveling in $\hat{z}$ direction

$$\vec{S} = c\cdot u\,\hat{z}$$

$$[\vec{S}] = E\,L^{-2}T^{-1}$$
$$[u] = E\,L^{-3}$$
$$[\vec{S}] = [u]\cdot \underbrace{L\,T^{-1}}_{[c]}$$

time avg

$$\langle \vec{S} \rangle = \frac{1}{2\mu_0}\left(\mathrm{Re}[\tilde{\vec{E}}\times\tilde{\vec{B}}^*]\right)$$ for some $\vec{k}, \omega$

momentum density

$$\vec{p}^{\,\text{fields}} = \mu_0\epsilon_0\,\vec{S} = \frac{1}{c^2}\cdot\vec{S} = \frac{1}{c}u\,\hat{z}$$

in direction of fields traveling, you carry some energy & momentum

$$\langle u \rangle = \frac{1}{2}\epsilon_0 E_0^2$$

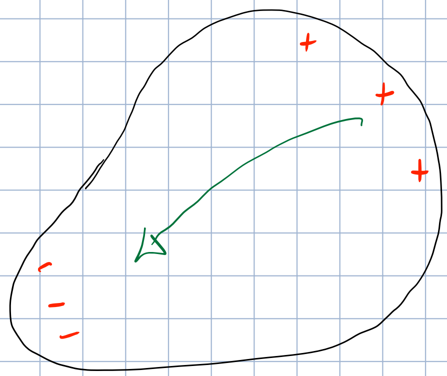
$$\langle \vec{S} \rangle = c\cdot\langle u \rangle\,\hat{z} = \frac{1}{2}c\,\epsilon_0 E_0^2\,\hat{z}$$

$$\langle \vec{p} \rangle = \frac{1}{c^2}\langle \vec{S} \rangle = \frac{1}{2c}\epsilon_0 E_0^2\cdot\hat{z}$$

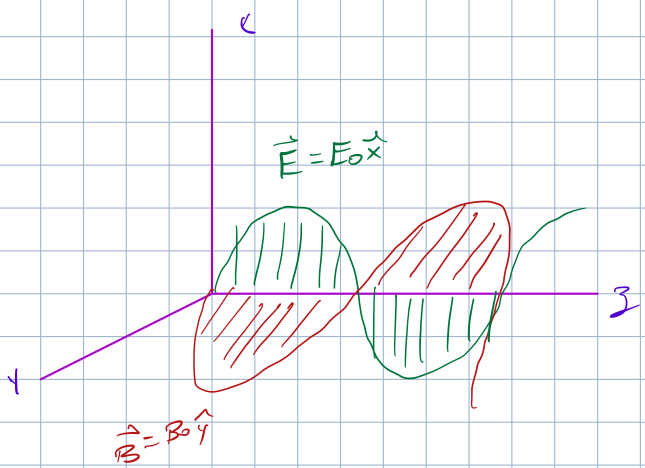
avg of cos is ½

dimensional analysis! ⟿ radiation pressure in notes

Polarization: defined as preferred direction



net polarization $\propto \vec{E}$



$\hat{n} = \cos\theta \cdot \hat{x} + \sin\theta \cdot \hat{y}$

Maxwell in Matter

$\vec{P}$ polarization
dipole/volume

$\vec{H}$ net magnetic field
extra $\vec{B}$ b/c of $\vec{M}$

$\vec{D}$ displacement
extra $\vec{E}$ b/c of $\vec{P}$

$\vec{M}$ magnetization
dipole/volume