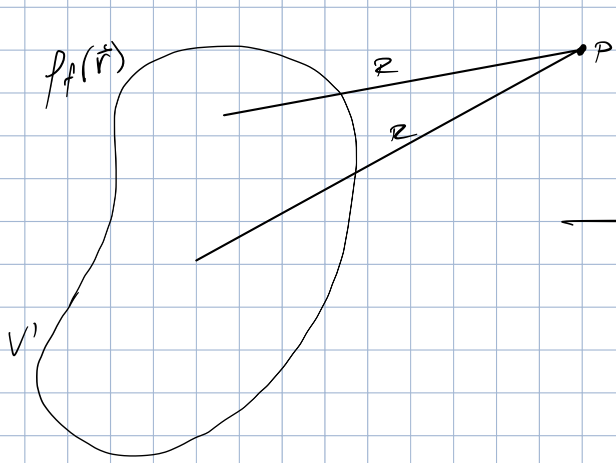
Electric Displacement

V produced by polarized object w/ nonzero charge



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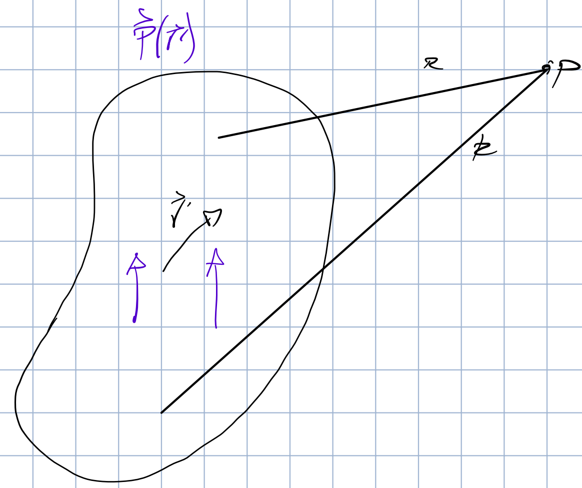


$$V_f(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho_f(\vec{r}')}{R}\,d\tau'$$

free

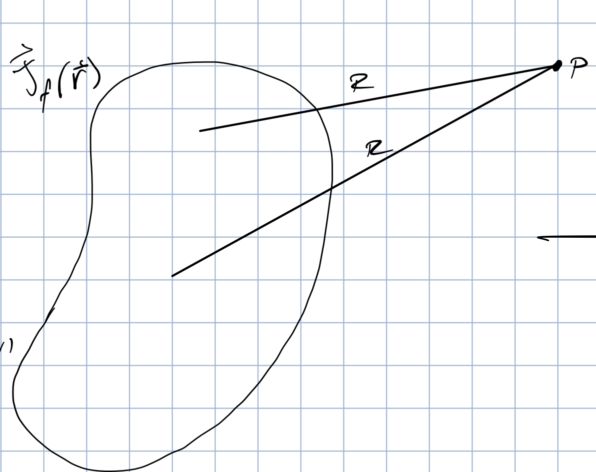
$$V_b(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left(\int \frac{\rho_b(\vec{r}')}{R}\,d\tau' + \int \frac{\sigma_b(\vec{r}')}{R}\,da'\right)$$

$$\rho_b(\vec{r}') = -\vec{\nabla}'\cdot\vec{P}(\vec{r}')$$
$$\sigma_b(\vec{r}') = \vec{P}(\vec{r}')\cdot\hat{n}'$$

bound - produced by polarization

$$\rho = \rho_f + \rho_b = \rho_f - \vec{\nabla}\cdot\vec{P}$$
$$\sigma = \sigma_f + \sigma_b = \sigma_f + \vec{P}(\vec{r})\cdot\hat{n}$$
$$\vec{E} = \vec{E}_f + \vec{E}_b$$

Auxiliary field H



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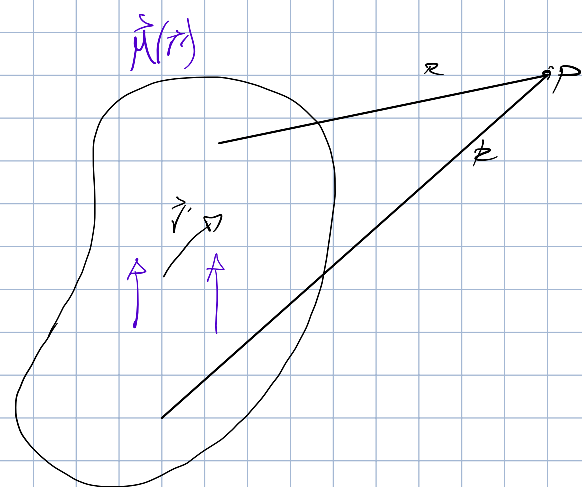


$$\vec{A}_f = \frac{\mu_0}{4\pi}\int \frac{\vec{J}_f(\vec{r}')}{R}\,d\tau$$

free

$$\vec{A}_b(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}_b(\vec{r}')}{R}\,d\tau + \frac{\mu_0}{4\pi}\int \frac{\vec{K}_b(\vec{r}')}{R}\,da'$$

$$\vec{J}_b(\vec{r}') = \vec{\nabla}' \times \vec{M}(\vec{r}')$$
$$\vec{K}_b(\vec{r}') = \vec{M}(\vec{r}') \times \hat{n}'$$

bound - result of magnetization

$$\vec{J} = \vec{J}_f + \vec{J}_b = \vec{J}_f + \vec{\nabla}\times\vec{M}$$

$$\vec{K} = \vec{K}_f + \vec{K}_b = \vec{K}_f + \vec{M}\times\hat{n}$$

$$\vec{B} = \vec{B}_f + \vec{B}_b$$

Electricity

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} = \frac{\rho_f}{\epsilon_0} + \frac{\rho_b}{\epsilon_0} = \frac{\rho_f}{\epsilon_0} - \frac{1}{\epsilon_0}\vec{\nabla}\cdot\vec{P}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P}$$

$$\epsilon_0\vec{\nabla}\vec{E} = \rho_f - \vec{\nabla}\vec{P}$$

$$\rho_f = \epsilon\vec{\nabla}E + \vec{\nabla}\vec{P} = \vec{\nabla}(\epsilon_0\vec{E} + \vec{P}) = \vec{\nabla}\cdot\vec{D}$$

Gauss: vacuum: free charge
general: free + bound charge

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0} \qquad \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\oint \vec{D}\cdot d\vec{a} = Q_{free,enc} \qquad \vec{\nabla}\cdot\vec{D} = \rho_f$$

Magnetism

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} = \mu_0\vec{J}_f + \mu_0\vec{J}_b = \mu_0\vec{J}_f + \mu_0\vec{\nabla}\times\vec{M}$$

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$$

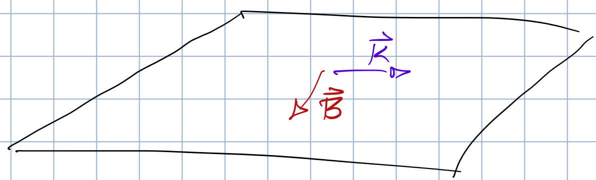
$$\frac{1}{\mu_0}\vec{\nabla}\times\vec{B} = \vec{J}_f + \vec{\nabla}\times\vec{M}$$

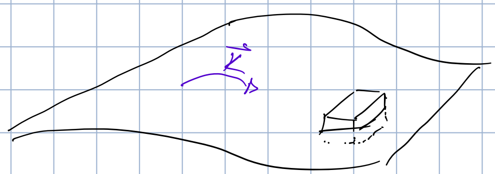
$$\vec{J}_f = \frac{1}{\mu_0}\vec{\nabla}\times\vec{B} - \vec{\nabla}\times\vec{M}$$
$$= \vec{\nabla}\times\left(\frac{1}{\mu_0}\vec{B} - \vec{M}\right)$$
$$= \vec{\nabla}\times\vec{H}$$

Ampere vacuum: free current
general: free + bound current

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$ $\qquad$ $\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}$

$\oint \vec{H}\cdot d\vec{l} = I_{free,enc}$ $\qquad$ $\vec{\nabla}\times\vec{H} = \vec{J}_{free}$





$\vec{\nabla}\cdot\vec{B} = 0$

$\oint \vec{B}\cdot d\vec{a} = 0$

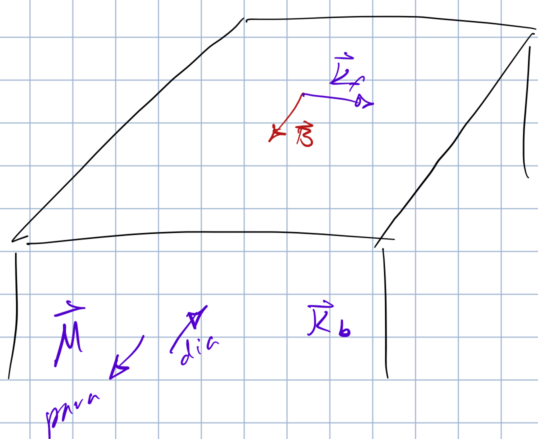
$B^{\perp}_{above} - B^{\perp}_{below} = 0$

$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$

$\oint \vec{B}\cdot d\vec{l} = \mu_0 \vec{K} l$

$B^{\parallel}_{above} - B^{\parallel}_{below} = \mu_0 K$

$\vec{B}^{\parallel}_{above} - \vec{B}^{\parallel}_{below} = \mu_0 \vec{K}\times\hat{n}$



$\vec{\nabla}\cdot\vec{B} = 0$

$B^{\perp}_{above} - B^{\perp}_{below} = 0$

$\vec{\nabla}\cdot\vec{H} = -\vec{\nabla}\cdot\vec{M}$

$H^{\perp}_{above} - H^{\perp}_{below} = -(M^{\perp}_{above} - M^{\perp}_{below})$

$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} = \mu_0(\vec{J}_f + \vec{J}_b)$

$B^{\parallel}_{above} - B^{\parallel}_{below} = \mu_0 (K_f + K_b)$ $\quad$ vector $\rightarrow$ $\times\hat{n}$

$\vec{\nabla}\times\vec{H} = \vec{J}_f$

$H^{\parallel}_{above} - H^{\parallel}_{below} = K_f$ $\quad$ vector $\rightarrow$ $\times\hat{n}$

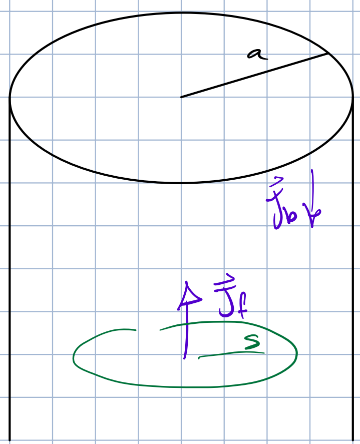
in vacuum:

$$B^{\perp}_{above} - B^{\perp}_{below} = 0$$
$$\vec{B}^{\parallel}_{above} - \vec{B}^{\parallel}_{below} = \mu_0 \vec{K} \times \hat{n}$$

in magnetized material:

$$B^{\perp}_{above} - B^{\perp}_{below} = 0$$
$$\vec{B}^{\parallel}_{above} - \vec{B}^{\parallel}_{below} = \mu_0 (\vec{K}_f + \vec{K}_b) \times \hat{n}$$
$$H^{\perp}_{above} - H^{\perp}_{below} = -(M^{\perp}_{above} - M^{\perp}_{below})$$
$$H^{\parallel}_{above} - \vec{H}^{\parallel}_{below} = \vec{K}_f \times \hat{n}$$

Example 6.2 long copper rod w/uniform current I



ϕ symmetric

① $\vec{B}$ inside due to current?

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$B s \cdot 2\pi = \mu_0 I \frac{\pi s^2}{\pi a^2}$$

$$\vec{B} = \mu_0 \frac{I}{2\pi a^2} s \hat{\phi}$$

$\vec{B}$ is not constant

$$\vec{H} = \frac{1}{\mu_0} \vec{B}$$

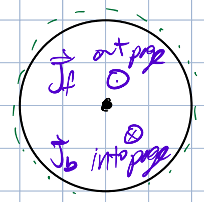
② copper magnetized by $\vec{B}$

copper diamagnetic, so $\vec{M}$ opposite $\vec{B}$

$$\vec{M} = -|\vec{M}| \hat{\phi}$$

$$\vec{J}_b = \nabla \times \vec{M}$$

$$\vec{K}_b = \vec{M} \times \hat{n} = -|\vec{M}|\,\hat{\phi} \times \hat{s} = |\vec{M}|\,\hat{s} \times \hat{\phi} = |\vec{M}|\,\hat{z} \qquad \uparrow \vec{K}_b$$



$\vec{K}_b$ out of page

(3) $$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$$

H inside rod

$$\oint \vec{H} \cdot d\vec{l} = I_{enc,f}$$

$$2\pi s H = I \cdot \frac{\pi s^2}{\pi a^2}$$

$$\vec{H} = I \frac{s}{2\pi a^2}\,\hat{\phi}$$

H outside rod

$\vec{M} = 0$ (only in material)

$$\vec{H} = \frac{1}{\mu_0}\vec{B}$$

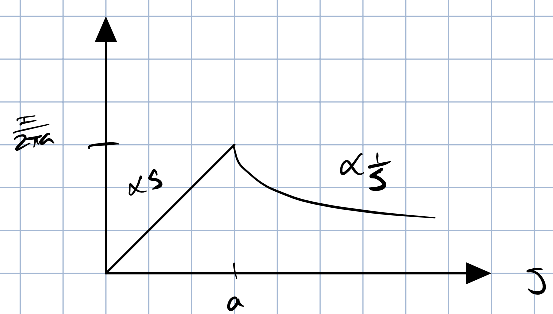
$$\vec{B} = \mu_0 \vec{H} = \frac{\mu_0 I}{2\pi s}\,\hat{s}$$

$$\oint \vec{H} \cdot d\vec{l} = I_{encl,f} = I$$

$$H\, 2\pi s = I$$

$$\vec{H} = \frac{I}{2\pi s}\,\hat{\phi}$$

Boundary Conditions



$\perp$ Components

$$H^\perp_{above} - H^\perp_{below} = \underset{0}{H^\perp_{ext}(s=a)} - \underset{0}{H^\perp_{in}(s=a)} = 0$$

$$M^\perp_{above} - M^\perp_{below} = \underset{0}{M^\perp_{ext}} - \underset{0}{M^\perp_{in}} = 0$$

$$\rightarrow H^\perp_{above} - H^\perp_{below} = -\left(M^\perp_{above} - M^\perp_{below}\right)$$

$\parallel$ components

$$H^{\parallel}_{above} - H^{\parallel}_{below} = H^{\parallel}_{ext}(s=a) - H^{\parallel}_{in}(s=a) = \frac{I}{2\pi a}\hat{\phi} - \frac{I}{2\pi a}\hat{\phi} = 0$$

$$\vec{K}_f \times \hat{n} = 0 \qquad \text{so } \vec{K}_f = 0$$

$$\vec{H}^{\parallel}_{above} - \vec{H}^{\parallel}_{below} = \vec{K}_f \times \hat{n}$$