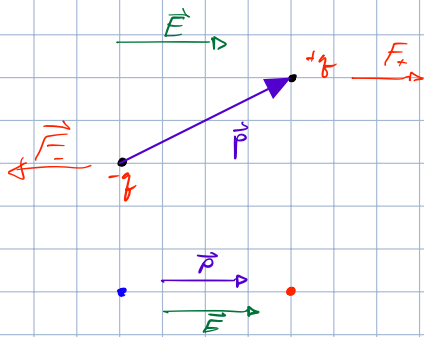


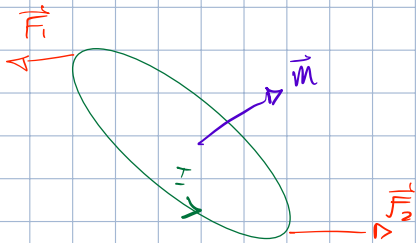
# Magnetization



net force:  $\vec{F}_+ + \vec{F}_- = 0$

net torque:  $\vec{\tau} = \vec{p} \times \vec{E}$

$\vec{p} \parallel \vec{E}$



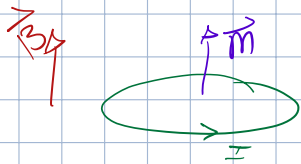
$\vec{F}_1 = I \cdot d\vec{l}_1 \times \vec{B}$   
 $\vec{F}_2 = I \cdot d\vec{l}_2 \times \vec{B}$        $d\vec{l}_1 = -d\vec{l}_2$

$\rightarrow \vec{F}_1 + \vec{F}_2 = 0$

net force:  $\vec{F} = 0$

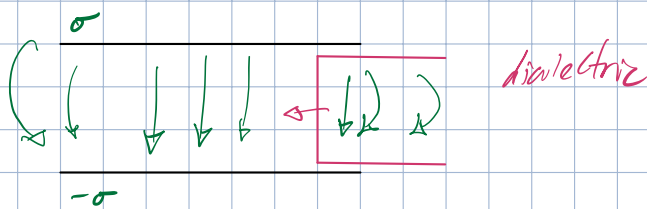
net torque:  $\vec{N} = \vec{m} \times \vec{B}$

$\vec{m} \parallel \vec{B}$



## Nonuniform $\vec{E}$

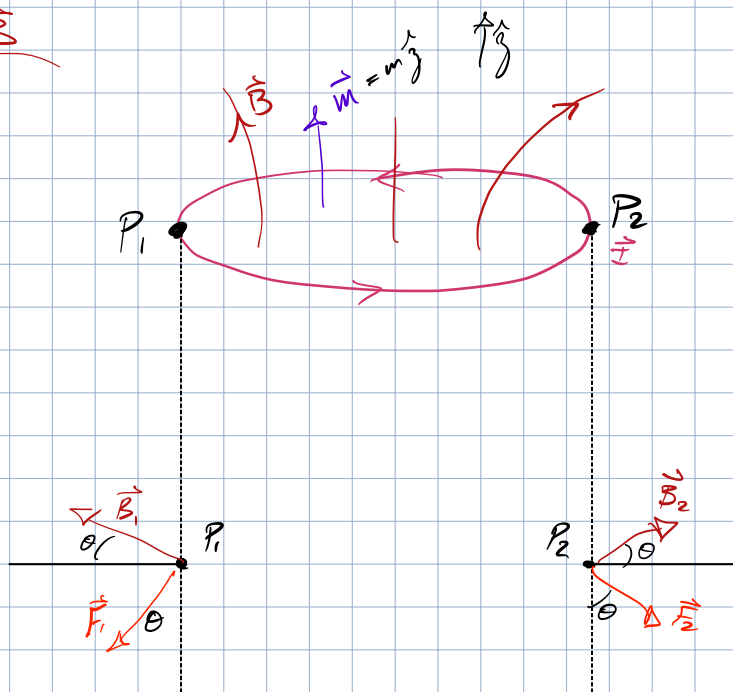
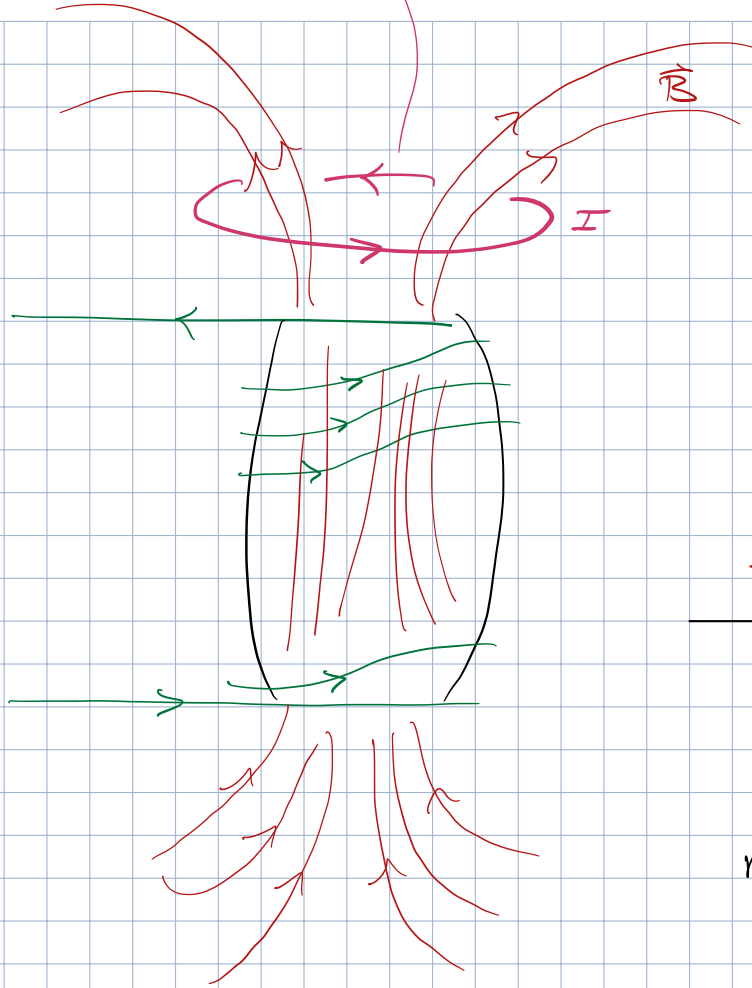
bring dielectric  $\rightarrow$  pulled into capacitor



get fringe fields

## Nonuniform $\vec{B}$

bring wire loop (w/ current I)  $\rightarrow$  pulled into solenoid



net force  $\downarrow$

$$F = \nabla(\vec{m} \cdot \vec{B})$$

$$\begin{aligned}
 |F_1| &= |F_2| = F \cos \theta = qvB_0 = (2\pi SI) \cdot B \cos \theta \\
 &= 2\pi SI \hat{j} \cdot \vec{B} \\
 &= \frac{d\vec{m}}{dt} \cdot \vec{B} = \frac{\partial}{\partial t}(\vec{m} \cdot \vec{B}) \quad \vec{m} = \pi s^2 I \hat{j} \\
 \vec{F} &= \nabla(\vec{m} \cdot \vec{B})
 \end{aligned}$$

## Non-conducting Materials

each atom has tiny magnetic dipole (spins, orbitals, ...)

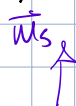
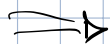
$\vec{E}$  fields polarize materials

$\vec{B}$  fields magnetize materials

atoms w/ unpaired  $e^-$



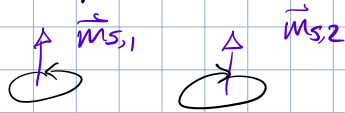
contributions by spin is much greater than those by orbital motion



$$\vec{m}_{s, \text{after}} - \vec{m}_{s, \text{before}} > 0$$

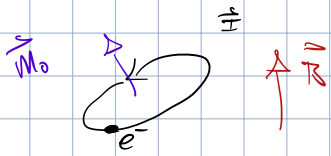
$\rightarrow$  "paramagnetism"

② atoms w/ paired  $e^-$



Spin contributions cancel

$\vec{m}$  is due to orbital motion of  $e^-$ 's



harder to tilt entire orbit

$\vec{B}$  affects orbital motion  $\rightarrow$  speed up/slow down



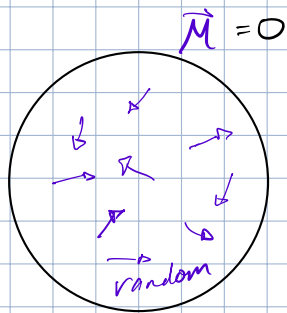
$e^-$  speed reduced,  $\vec{m}$  reduced



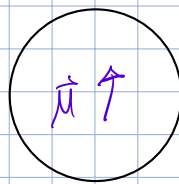
$e^-$  speed increased,  $\vec{m}$  increased

$\vec{m}$  in  $\vec{B}$  direction is reduced opposite to paramagnetism

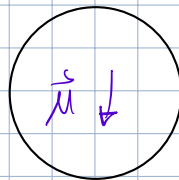
"diamagnetism"



apply  $\vec{B}$

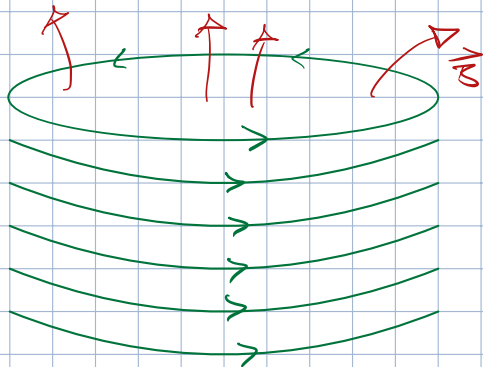


paramagnetism

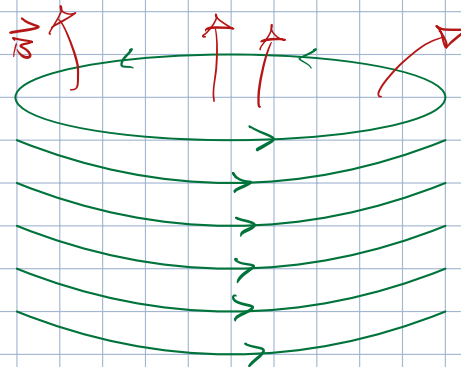


diamagnetism

paramagnetic material



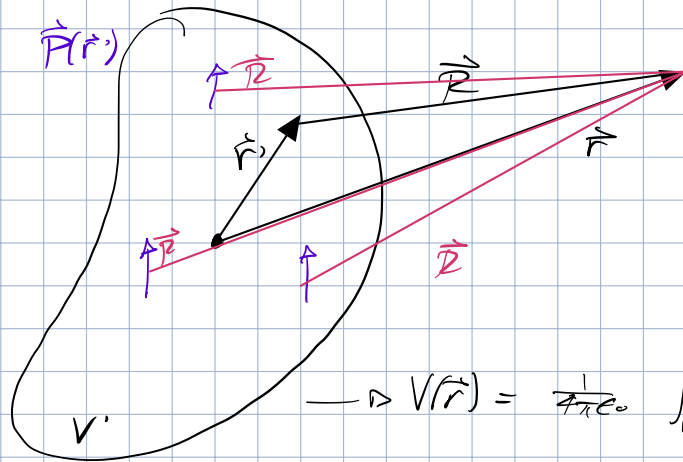
diamagnetic material



$\vec{P}$  = polarization  
= electric dipole per unit volume

$\vec{M}$  = Magnetization  
= magnetic dipole per unit volume

# Polarized Material

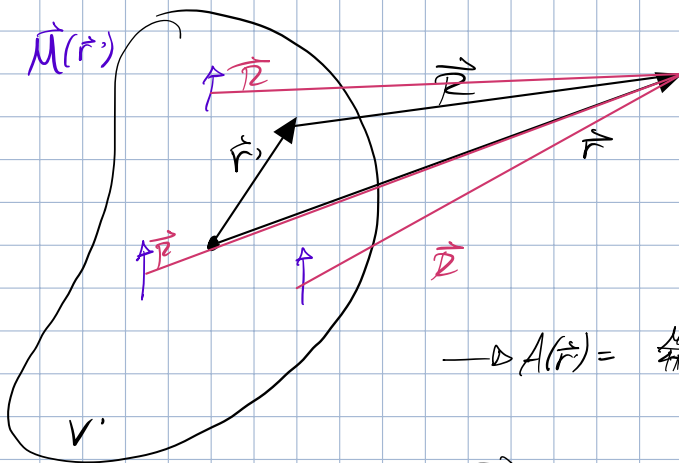


$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{V'} \frac{\vec{P}(\vec{r}') \cdot \vec{e}}{R^2} d\tau'$$

$$\rightarrow V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{V'} \frac{\rho_{\text{bound}}(\vec{r}')}{R} d\tau' + \frac{1}{4\pi\epsilon_0} \oint_S \frac{\sigma_{\text{bound}}(\vec{r}')}{R} d\vec{a}'$$

$$\rho_{\text{bound}}(\vec{r}') = -\vec{\nabla}' \cdot \vec{P}(\vec{r}')$$

$$\sigma_{\text{bound}}(\vec{r}') = \vec{P}(\vec{r}') \cdot \hat{n}'$$



$$A(\vec{r}) = \frac{\mu_0}{4\pi} \int_{V'} \frac{M(\vec{r}') \times \vec{e}}{R^2} d\tau'$$

$$\rightarrow A(\vec{r}) = \frac{\mu_0}{4\pi} \int_{V'} \frac{\vec{J}_{\text{bound}}(\vec{r}')}{R} d\tau' + \frac{\mu_0}{4\pi} \oint_S \frac{\vec{K}_{\text{bound}}(\vec{r}')}{R} d\vec{a}'$$

$$\vec{J}_{\text{bound}}(\vec{r}') = \vec{\nabla}' \times M(\vec{r}')$$

$$\vec{K}_{\text{bound}}(\vec{r}') = M(\vec{r}') \times \hat{n}'$$