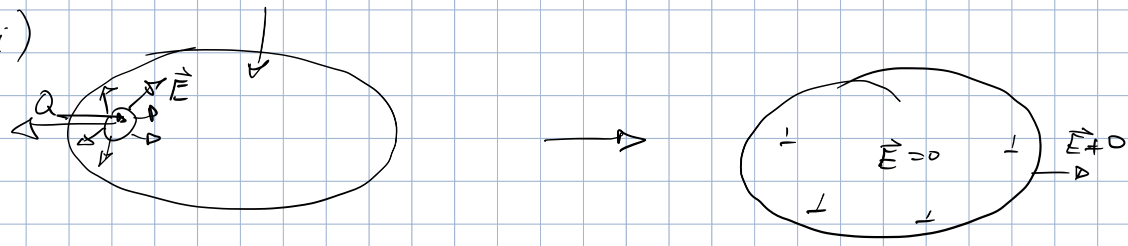
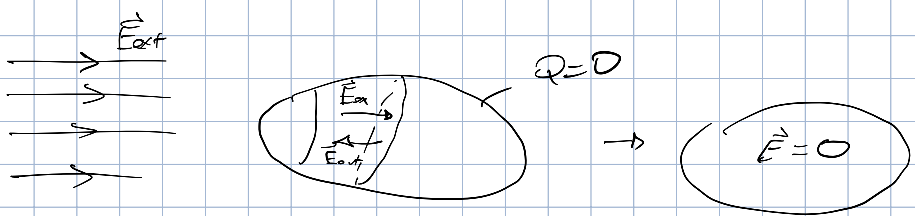
Conductors (G. 2.5)

i)



$\vec{E}$ in conductor is 0

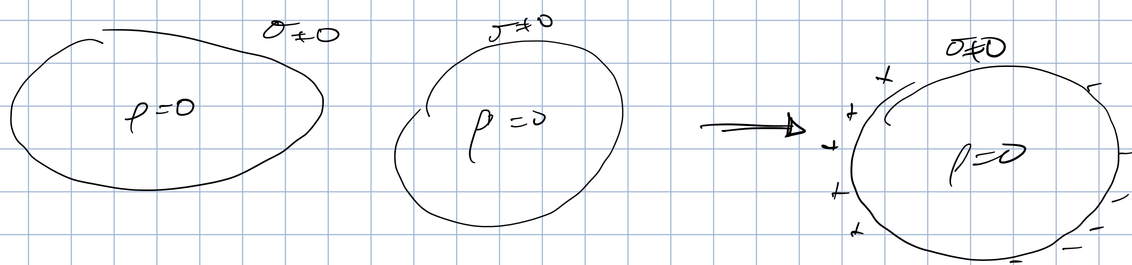
ii)



iii) $\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0}$

$\vec{E}=0 \to \rho=0$

iv)



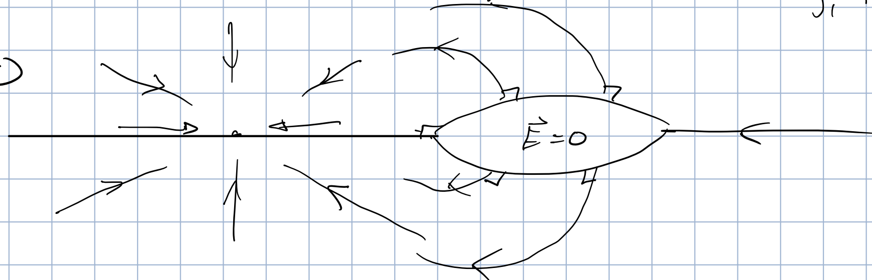
v)

equipotential

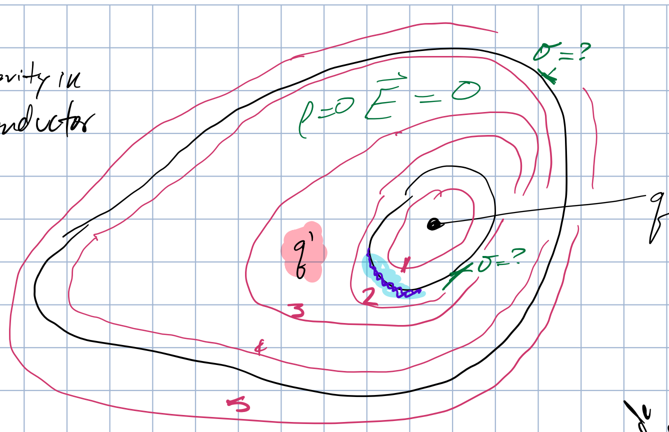
$$\Delta V = V_2 - V_1 = \int_1^2 (\nabla V)\cdot d\vec{l}$$

$$= \int_1^2 (\overset{=0}{\vec{E}})\cdot d\vec{l} = 0$$

#1 $q<0$



#2 cavity in conductor



hollow

gaussian surface

inside cavity $\oint \vec{E}_1 \cdot d\vec{a} = \frac{Q_{encl.}}{\epsilon_0} = \frac{q}{\epsilon_0}$

$\vec{E}_1 \neq 0$

inside conductor just outside cavity $\oint \vec{E}_2 \cdot d\vec{a} = \frac{q + q_{cond.}}{\epsilon_0}$

conductor could have surface charge

$\vec{E}_2 = 0 \rightarrow \frac{q + q_{cond}}{\epsilon_0} = 0 \rightarrow q_{cond} = -q$

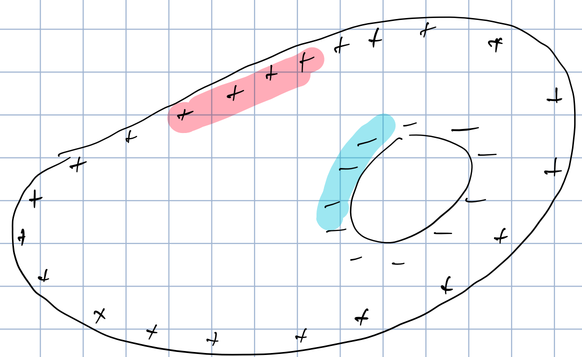
further out $\oint \vec{E}_3 \cdot d\vec{a} = \frac{q + q_{cond.} + q'_{cond}}{\epsilon_0} \stackrel{?}{=} 0$ $\quad \vec{E}_3 = 0$

$\vec{E}_2 = \vec{E}_3 = \vec{E}_4 = \vec{E}_{cond}$

$\vec{E}_1 = \vec{E}_{cavity}$

just outside conductor $\oint \vec{E}_5 \cdot d\vec{a} = \frac{q}{\epsilon_0}$ (neutral conductor, net $Q=0$, $q_{cond}=0$)

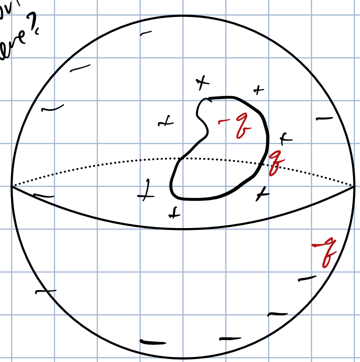
$\vec{E}_{out} = \vec{E}_5 \neq 0$



q_{cond}

$-q_{cond}$

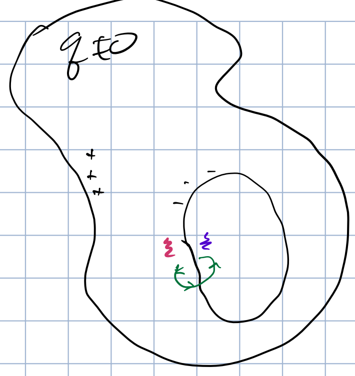
how about sphere?



uniformally distributed charge

$\sigma =$ constant

not neutral conductor



$\oint \vec{E}\cdot d\vec{l} = 0$ — always true

from before

$\oint \vec{E}_{cavity}\cdot d\vec{l} + \cancel{\oint \vec{E}_{cond}\cdot d\vec{l}} = 0$

$\oint \vec{E}_{cavity}\cdot d\vec{l} = 0 \rightarrow \vec{E}_{cavity} = 0$

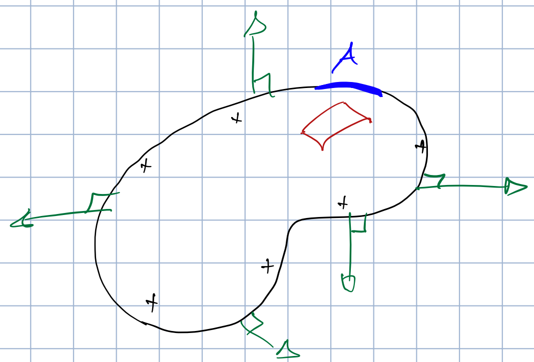
$\oint \vec{E}\cdot d\vec{a} = \frac{q_{encl}}{\epsilon_0} = \cancel{\oint \vec{E}_{cavity}\cdot d\vec{a}} + \cancel{\oint \vec{E}_{cond.}\cdot d\vec{a}}$

$\longrightarrow q_{encl.} = 0$

$\sigma^{(conductor)}_{inner\,wall} = 0$

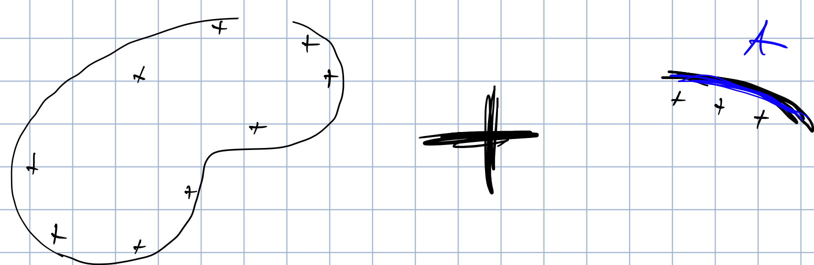
but there's some inside q → where does it cancel? outerwall!

Surface Charge & Force on a Conductor

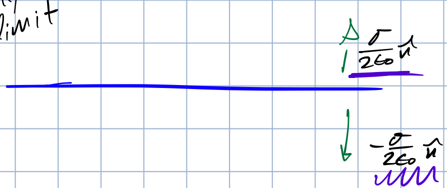


from before: $\vec{E}_{above} - \underbrace{\vec{E}_{below}}_{\to 0} = \frac{\sigma}{\epsilon_0}\cdot\hat{n}$

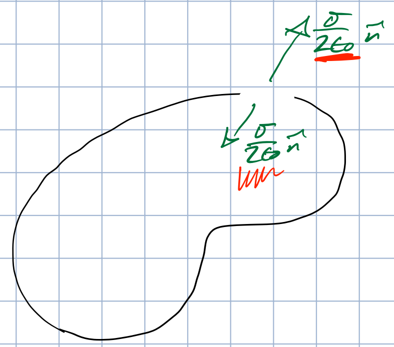
before: $\vec{E}_{outside} = \frac{\sigma}{2\epsilon_0}\hat{n}$

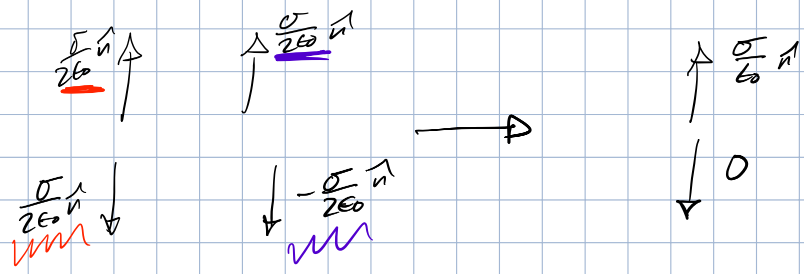


infinitely large limit



$\vec{E}_{patch} = \vec{E}_A = \frac{\sigma}{2\epsilon_0}\hat{n}$



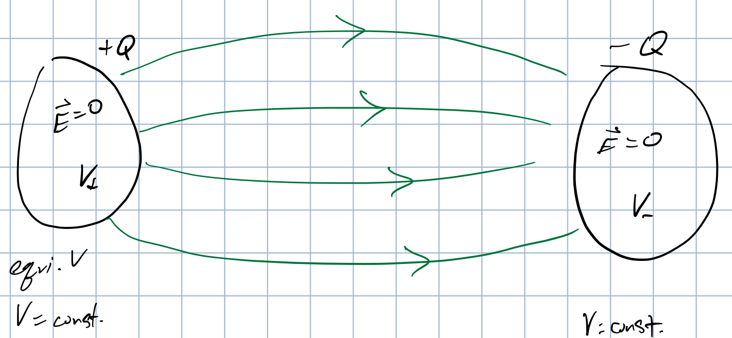


$$\vec{F}_{patch} = Q_{patch} \cdot \vec{E}_{other}$$

$$F_p = \sigma A \frac{\sigma}{\epsilon_0}\hat{n} = A \frac{\sigma}{2\epsilon_0}\hat{n}$$

pressure force per area

Capacitors

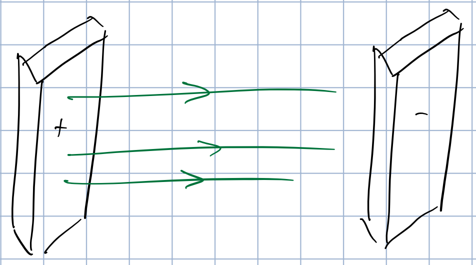


$$\Delta V = V_+ - V_-$$
$$= -\int_-^+ \vec{E} \cdot d\vec{l}$$

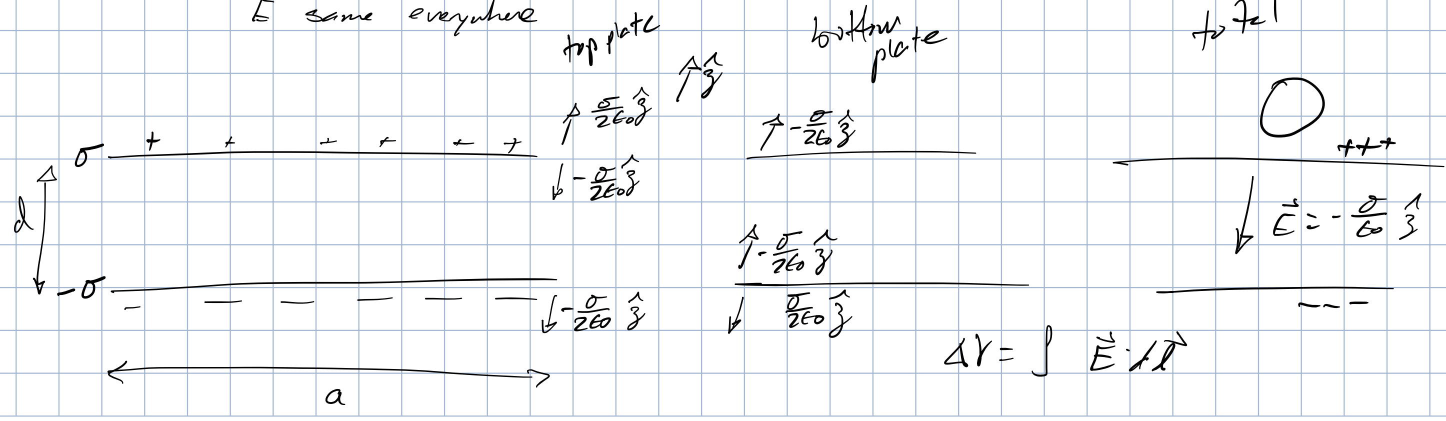
$$|\vec{E}| \propto Q$$

$$\Delta V \propto Q$$

$$\frac{Q}{\Delta V} = \text{const.} \equiv C \quad (\text{capacitor})$$



if $a \gg d$ $\longrightarrow$ use same arg. of infinite plane
$\vec{E}$ same everywhere



$$\Delta V = \int \vec{E} \cdot d\vec{l}$$

$= E \cdot d$

$Q = \sigma A$

$C = \frac{Q}{\Delta V} = \frac{Q}{E \cdot d} \longrightarrow$ const

$E = \frac{\sigma}{\epsilon_0} = \frac{Q/A}{\epsilon} = \frac{Q}{A\epsilon_0}$

C depends only on geometry