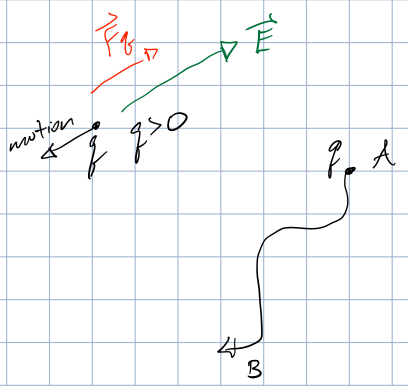
Work & Energy (G 2.4)



$$\vec{F}_w = -\vec{F}_q \quad (\text{if } q>0)$$

$$W_{A\to B} = \int_A^B \vec{F}_w \cdot d\vec{l}$$

$$= -\int_A^B \vec{F}_q \cdot d\vec{l} \qquad q\cdot\vec{E} = -q\nabla V$$

$$= q\int_A^B (\nabla V)\, d\vec{l}$$

$$W_{A\to B} = q\big(V(B) - V(A)\big)$$

$V_A = 0 \quad (\infty)$

$W_{\infty\to B} = qV_B$

$$W = qV$$

i) no $\vec{E}$, all external

q_1

$\vec{E} = 0$

$U^{(1)}_{\infty\to\vec{r}_1} = 0$ $\qquad$ U ~ total work

ii)

q_2

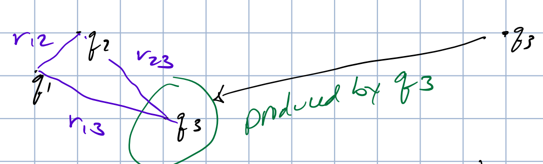
q_1

q_1 makes $\vec{E}$ on q_2 $\to$ $\vec{E}\neq 0$ & $V\neq 0$

$$U^{(2)}_{\infty\to\vec{r}_2} = q_2 \cdot V(\vec{r}_2)$$

$$= q_2 \cdot \frac{q_1}{4\pi\epsilon_0 |\vec{r}_2 - \vec{r}_1|}$$

iii)

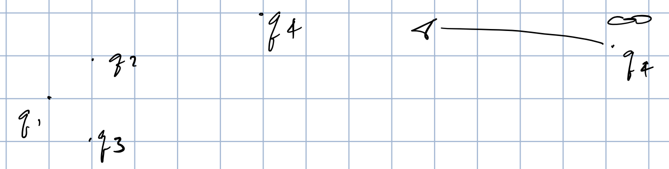


$$V = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1}{r_{13}} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_{23}}$$

$$U^{(3)}_{\infty \to \vec{r}_3} = q_3 \cdot V = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$\text{total } U = \cancel{U^{(1)}}^{\,0} + U^{(2)} + U^{(3)}$$

iv)



$$U^{(4)} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_4}{r_{14}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right)$$

$$U = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{3} \sum_{j=1}^{4} \frac{q_i q_j}{r_{ij}}$$

n charges ...

$$U = \sum_{k=1}^{n} U^{(k)} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n-1} \sum_{j>i}^{n} \frac{q_i \cdot q_j}{r_{ij}}$$

$r_{ij} = r_{ji}$

$$\frac{1}{r_{ij}} = \frac{1}{2}\left(\frac{1}{r_{ij}} + \frac{1}{r_{ij}}\right) = \frac{1}{2}\left(\frac{1}{r_{ij}} + \frac{1}{r_{ji}}\right)$$

$$\longrightarrow U = \frac{1}{4\pi\epsilon_0} \cdot \frac{1}{2} \cdot \sum_{i=1}^{n-1} \sum_{\substack{j=1 \\ i \neq j}}^{n} \frac{q_i \cdot q_j}{r_{ij}}$$

$$= \frac{1}{2} \cdot \sum_{i=1}^{n \; - i\neq j} q_i \cdot \sum_{j=1}^{n} \frac{1}{4\pi\epsilon_0} \frac{q_j}{r_{ij}}$$

energy of this configuration

potential from all $q_j \neq q_i$ produced by q_j

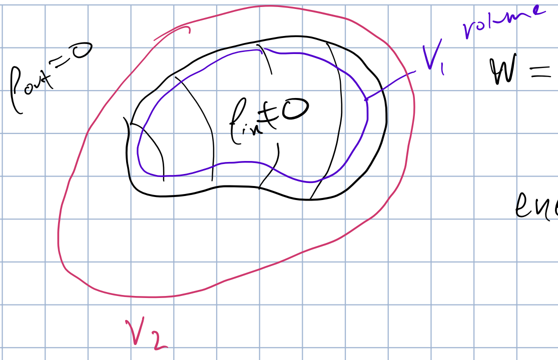
without q_i, what's the potential?

$$W = U = \frac{1}{2} \sum_{i=1}^{n} q_i \cdot V(\vec{r}_i)$$

in 3D: $W = U = \frac{1}{2} \int V \cdot \rho \, d\tau$

in 2D: $W = U = \frac{1}{2} \int V \cdot \sigma \, da$

in 1D: $W = U = \frac{1}{2} \int V \cdot \lambda \, dl$



$$W = U = \frac{1}{2}\int_{V_1} V\rho\, d\tau = \frac{1}{2}\int_{V_2} V\cdot\rho\, d\tau$$

energy indpt. of configuration

$\rho \cdot V$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\rho\cdot V = \underbrace{\epsilon_0 \cdot (\vec{\nabla}\cdot\vec{E})V}$$

$$\vec{\nabla}(V\cdot\vec{E}) = (\vec{\nabla}\cdot\vec{E})\cdot V + \vec{E}\cdot(\vec{\nabla}V) \qquad -\vec{\nabla}V = \vec{E}$$

vector calc

$$= (\vec{\nabla}\cdot\vec{E})\cdot V - \vec{E}^2$$

$$\longrightarrow (\vec{\nabla}\cdot\vec{E})\cdot V = \vec{E}\cdot\vec{E} + \vec{\nabla}(V\cdot\vec{E}) \qquad \cdot\,\epsilon_0$$

$$\underbrace{\epsilon_0(\vec{\nabla}\cdot\vec{E})\cdot V} = \epsilon_0\cdot\vec{E}\cdot\vec{E} + \epsilon_0\cdot\vec{\nabla}(V\cdot\vec{E})$$

$$W = U = \frac{1}{2}\int \underbrace{\rho V}\, d\tau$$

$$= \frac{1}{2}\epsilon_0\int \vec{E}^2 d\tau + \frac{1}{2}\epsilon_0 \underbrace{\int \vec{\nabla}(V\cdot\vec{E})\, d\tau}_{\int \vec{\nabla}\cdot(V\vec{E})\,d\tau = \oint_S (\vec{E}\cdot V)\, d\vec{a}}$$

$$= \frac{\epsilon_0}{2}\int \vec{E}^2 d\tau + \frac{\epsilon_0}{2}\oint_S (\vec{E}\cdot V)\, d\vec{a}$$

$\rho_{out} = 0$, $E_{out} \neq 0$ necessarily

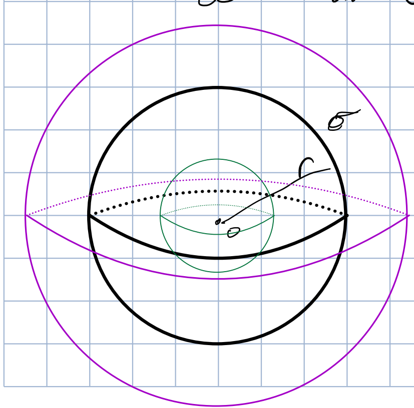
$\propto r^{-2}$ $\propto r^{-1}$ $\propto r^{2}$

$\propto r^{-3}$ $\propto r^{-1}$

grows

Shrinks

with all space: $$\boxed{U = W = \frac{\epsilon_0}{2}\int_{\text{all space}} \vec{E}^2\, d\tau}$$



thin shell

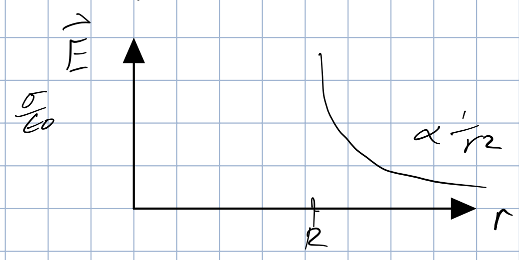
$r<R$: $\vec{E}_{in} = 0$

$r>R$: $\vec{E}_{above} = -\vec{E}_{below} \rightarrow \vec{E}_{out} = \frac{\sigma}{\epsilon_0}\hat{n} = \frac{\sigma}{\epsilon_0}\hat{r}$

$V_{in} = const$

$V_{out} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\sigma \cdot 4\pi R^2}{r}$

$$U = W = \frac{1}{2}\int V \cdot \sigma\, da$$



Use or to get V