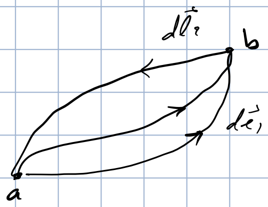
Electric Potential

ρ

no symmetry → V

symmetry, Gauss's Law → $\vec{E}$

a)



$$0 = \oint \vec{E}\cdot d\vec{l} = \int_A^B \vec{E}\cdot d\vec{l}_1 + \int_B^A \vec{E}\cdot d\vec{l}_2 = \int_A^B \vec{E}\cdot d\vec{l}_1 - \int_A^B \vec{E}\cdot d\vec{l}_2$$

$$\int_{A,\ \text{path 1}}^{B} \vec{E}\cdot d\vec{l} = \int_{A,\ \text{path 2}}^{B} \vec{E}\cdot d\vec{l}$$

b) $$\int_A^B \vec{E}\, d\vec{l} = -\int_A^B \frac{(\vec{\nabla} V)}{dV}\, d\vec{l} = -\int_A^B dV = -(V_A - V_B)$$

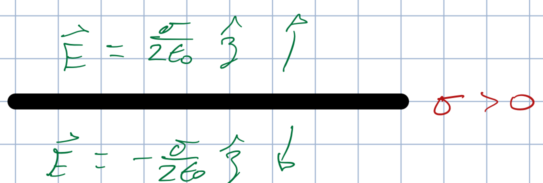
c) $$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{l}$$

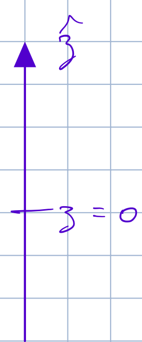
$$\begin{matrix}(V_B + c)\\(V_A + c)\end{matrix} \quad V_B + c - V_A - C = V_B - V_A$$

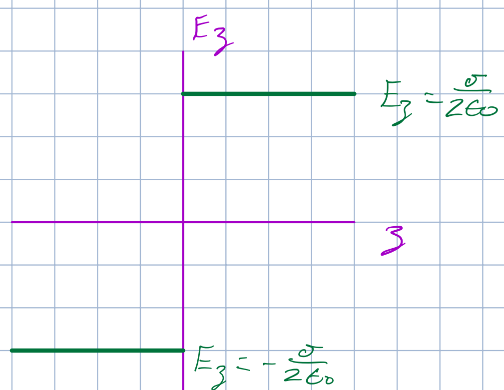
$$\longrightarrow V_B = -\int_A^B \vec{E}\cdot d\vec{l} + V_A$$

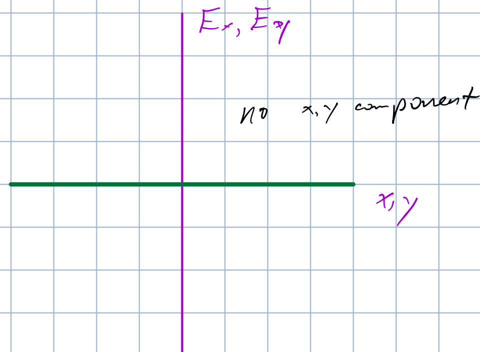
can set reference point to 0

Boundary Conditions (G 2.3.5)







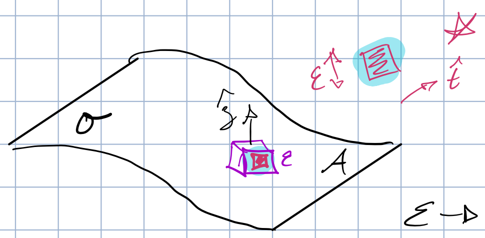


$$E_T = E_\perp = E_x = E_y = 0$$

tangent to surface

$$\Delta E_z = \frac{\sigma}{\epsilon_0}$$

$$\Delta E_x = \Delta E_y = 0$$



$\varepsilon \to 0$

charge $= \sigma \cdot A$

Gauss Law

$$\oint \vec{E}\cdot d\vec{a} = \frac{\sigma \cdot A}{\epsilon_0} = \vec{E}_{above}\cdot A\cdot\hat{z} - \vec{E}_{below}\cdot A\cdot\hat{z} + \cancel{\int_{side} \vec{E}_{side}\cdot d\vec{a}} \to 0 \text{ b/c } \varepsilon \to 0$$

$$\longrightarrow \frac{\sigma\cdot\cancel{A}}{\epsilon_0} = (\vec{E}_{above} - \vec{E}_{below})\cancel{A}\cdot\hat{z}$$

$$\frac{\sigma}{\epsilon_0} = E_{z,above} - E_{z,below}$$

$$= E_{\perp,above} - E_{z,below}$$

$$\oint \vec{E}\cdot d\vec{l} = 0 = \int_{below} \vec{E}\cdot d\vec{l} + \int_{above} \vec{E}\cdot d\vec{l} + \int_{side} \vec{E}\cdot d\vec{l}$$

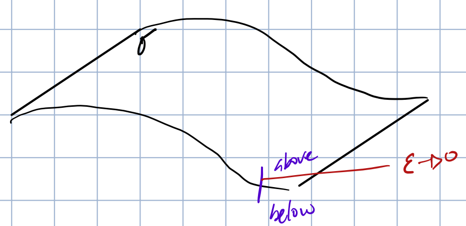
$d\vec{l} = dl\,\hat{t}$ $\quad$ $d\vec{l} = -dl\,\hat{t}$ $\quad$ $\varepsilon \to 0$

$$= l\cdot E^{\parallel}_{below} - l\cdot E^{\parallel}_{above} = 0$$

$$E_{below} = E_{above}$$

$$\vec{E}_{above} - \vec{E}_{below} = \frac{\sigma}{\epsilon_0}\cdot\hat{n}$$

$$\frac{\partial V_{above}}{\partial n} - \frac{\partial V_{below}}{\partial n} = \frac{\sigma}{\epsilon_0}$$

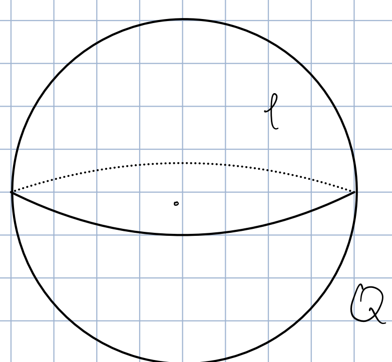


V just above plane

$$V_{above} - V_{below} = -\cancel{\int_{below}^{above} \vec{E}\cdot d\vec{l}} \to 0$$

$$V_{above} = V_{below}$$

$\boxed{\text{potential always continuous}}$



$\rho = \text{constant}$

Gauss $\to$ E_{in}, E_{out}

$V_{in} = (\quad) + C$

$V_{out} = (\quad) + C$

$V_\infty \to 0 \longrightarrow V_{out} = (\quad) + 0$

$$V_{in}(r=a) = V_{out}(r=a)$$

$$\Downarrow$$

$$V_{in} + C = V_{out}$$