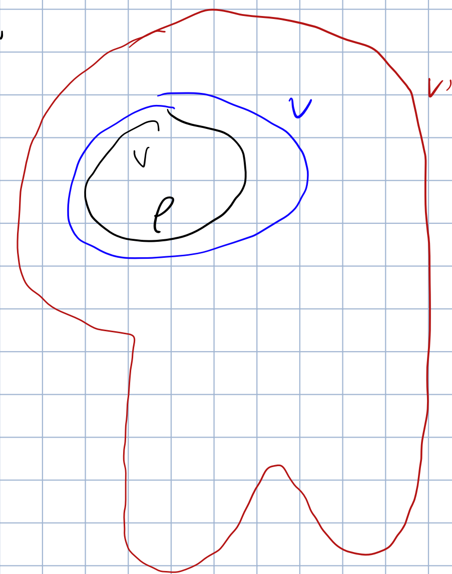
Divergence of $\vec{E}$ fields

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enclosed}}{\epsilon_0} = \frac{1}{\epsilon_0} \int_V \rho \, d\tau = \frac{1}{\epsilon_0} \int_{V'} \rho \, d\tau'$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$\Phi_1 = \Phi_2 = \Phi_3$



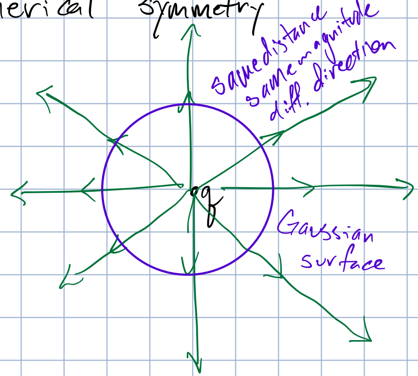
Gauss's Surface

positive area vector points out

① Figure out direction

② Construct surface

Spherical symmetry



$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot \hat{r}}$$

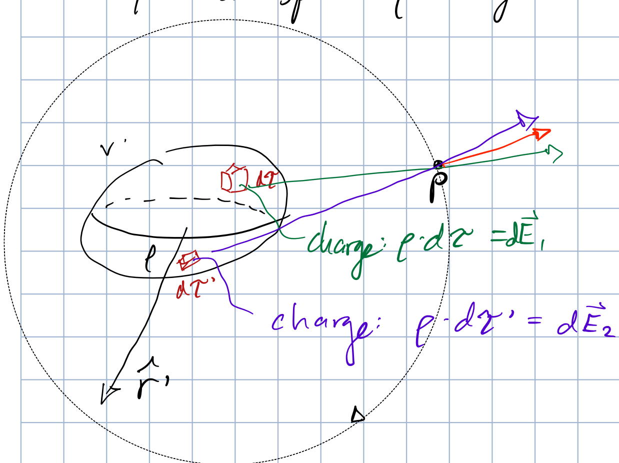
$$\oint \vec{E} \cdot d\vec{a} = \oint E\hat{r} \cdot da \cdot \hat{r} = \oint E \cdot da = E \oint da = E \cdot 4\pi r^2$$

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl.}}{\epsilon_0} \longrightarrow E \cdot 4\pi r^2 = \frac{Q_{encl}}{\epsilon_0}$$

$$\longrightarrow E = \frac{1}{4\pi\epsilon_0} \frac{Q_{encl}}{r^2}$$

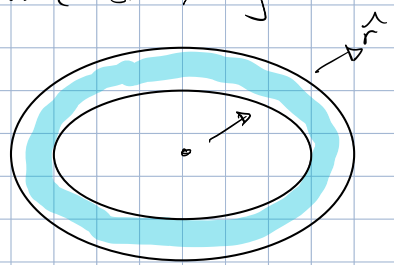
$$\boxed{\vec{E} = E \cdot \hat{r}}$$

say we have sphere w/ ρ charge



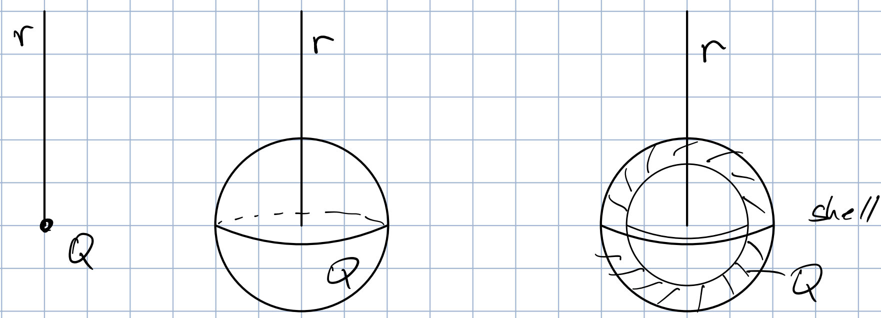
$d\vec{E}_1 + d\vec{E}_2 = \boxed{} \; \hat{r}$ radially

every opposite dτ will cancel out so only radial component survives look @ shell/ring



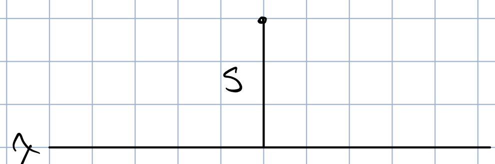
constant ρ

even w/ density as f^n of radius $\rho(r)$ $\rightarrow$ other sides will cancel b/c it's indpt. of θ & ϕ



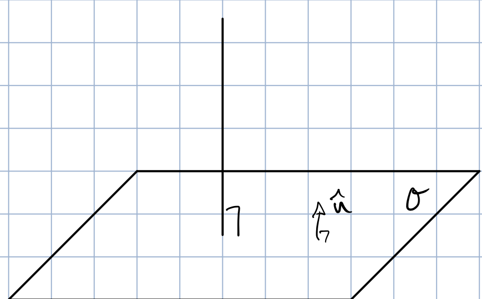
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$$

outside $\quad E \propto r^{-2}$



$$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2\lambda}{s} \cdot \hat{s}$$

$E \propto r^{-1}$



(infinite plane)

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{n}$$

$\hat{n}$ – normal to surface

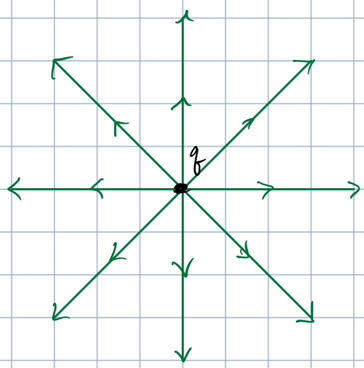
$E \propto \#$

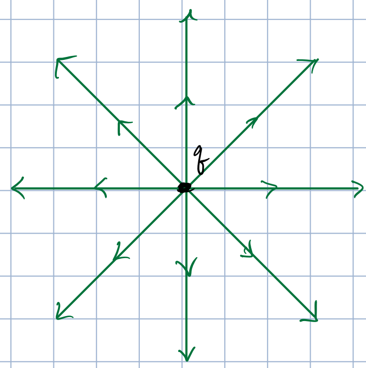
Noether

Symmetry $\longleftrightarrow$ conservation

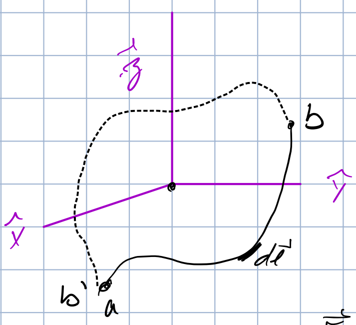
(Griffiths chapter & section)

Curl of $\vec{E}$ fields (G. 2.2.4)





$$\vec{\nabla}\times\vec{E} = 0$$



$$d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,\hat{\phi}$$

$$\oint \vec{E}\cdot d\vec{l} = \frac{1}{4\pi\epsilon_0}\, q \int_a^b \frac{1}{r^2}\,dr$$

$$= \frac{1}{4\pi\epsilon_0}\, q \left[-\frac{1}{r}\right]_a^b$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \qquad = \frac{1}{4\pi\epsilon_0}\, q \left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

$$\int_S (\vec{\nabla}\times\vec{E})\cdot d\vec{a} = \oint \vec{E}\cdot d\vec{l} \qquad \oint_a^{b'} \vec{E}\cdot d\vec{l} = 0$$

for any surface $\vec{\nabla}\times\vec{E} = 0$