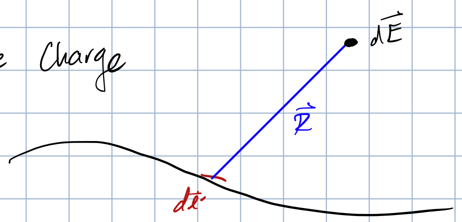
Principle of Superposition: $F_{Total} = Q \cdot E_{Total}$

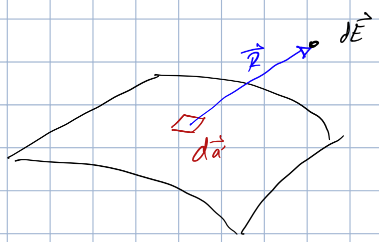
Line Charge



$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda}{R^2} \cdot \hat{R} \cdot dl'$$

λ - density

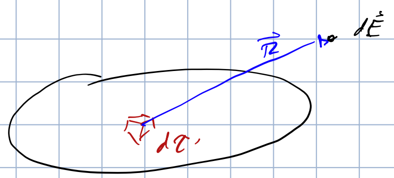
$$\vec{E} = \int d\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \int \frac{\lambda}{R^2} \hat{R} \cdot dl'$$



$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\sigma}{R^2} \cdot \hat{R} \cdot da'$$

σ - density

$$\vec{E} = \int d\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma}{R^2} \hat{R}\, da'$$



$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\rho}{R^2} \cdot \hat{R} \cdot d\tau'$$

$$\vec{E} = \int d\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \int \frac{\rho}{R^2} \hat{R}\, d\tau'$$

Electric Field Lines

extend to infinity

begin on positive $\rightarrow$ end on negative
don't terminate midair

density of field lines $\propto$ magnitude of field

Flux Through Surface

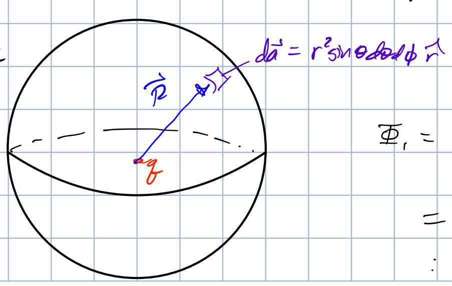
$$d\Phi_E = \vec{E} \cdot d\vec{a}$$

$$\Phi_E = \int_S \vec{E} \cdot d\vec{a} = \oint_S \vec{E} \cdot d\vec{a}$$

closed surface

net flux Φ_E: measure of # of field lines passing thru surface

closed surface



$$\Phi_1 = \oint \vec{E} \cdot d\vec{a}$$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \underline{\hat{r}} \cdot \sin\theta\, d\theta\, d\phi\, \underline{\hat{r}}$$

1

$$\Phi_1 = \frac{q}{\epsilon_0}$$

$\Phi_{Tot} = \frac{Q}{\epsilon_0}$ Q - total charge enclosed

Gauss's Law

integral form

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0} = \frac{1}{\epsilon_0}\int_V \rho \, d\tau$$

Differential form

divergence thm: $\int_V (\vec{\nabla}\cdot\vec{E})\, d\tau = \oint_S \vec{E}\cdot d\vec{a}$

$$= \int_V \frac{\rho}{\epsilon_0}\, d\tau$$

$$\cancel{\int_V} (\vec{\nabla}\cdot\vec{E})\, \cancel{d\tau} = \cancel{\int} \frac{\rho}{\epsilon_0}\, \cancel{d\tau}$$

$$\longrightarrow \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla}\cdot\vec{E} = \vec{\nabla}\cdot\left(\frac{1}{4\pi\epsilon_0}\int \frac{\rho}{\mathscr{r}^2}\hat{\mathscr{r}}\, d\tau'\right)$$

$$= \frac{1}{4\pi\epsilon_0}\int_V \vec{\nabla}\cdot\left[\frac{\rho(\vec{r}')}{\mathscr{r}^2}\underbrace{\hat{\mathscr{r}}}_{?}\right] d\tau'$$

$$= \frac{1}{4\pi\epsilon_0}\int_V \underbrace{\vec{\nabla}\cdot\left[\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right]}_{\text{prev. lecture}} \rho(\vec{r}')\, d\tau'$$

$$= \frac{1}{4\pi\epsilon_0}\int_V 4\pi\,\delta^3(\vec{\mathscr{r}})\,\rho(\vec{r}')\, d\tau'$$

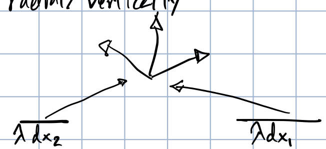
$$= \frac{1}{\epsilon_0}\int_{\text{all space}} \delta^3(\vec{r}-\vec{r}')\,\rho(\vec{r}')\, d\tau'$$

contribution outside of $\rho(\vec{r}')$ is 0

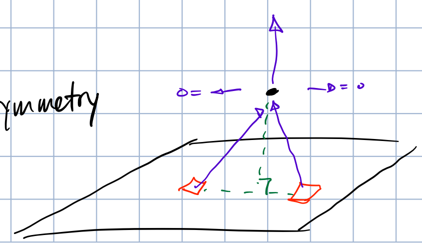
$$= \frac{1}{\epsilon_0}\rho(\vec{r})$$

Cylindrical Symmetry

no horizontal contribution, only radial/vertically



Plane Symmetry



① Direction of field @ P
② Magnitude of field

only $\hat{z}$ direction lives