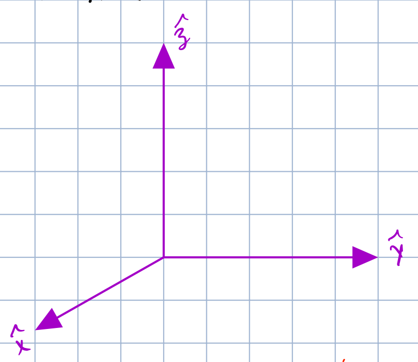
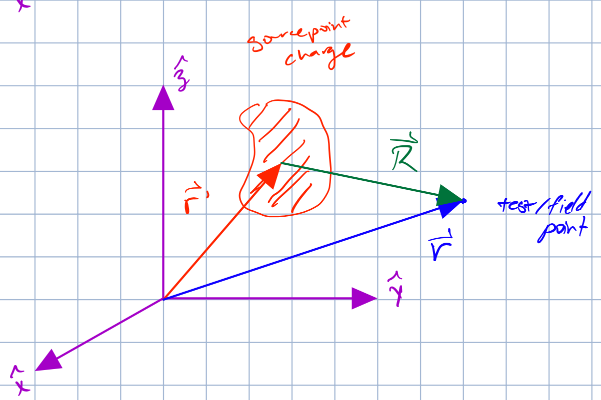
Chp. 1.1, 1.4, 1.2

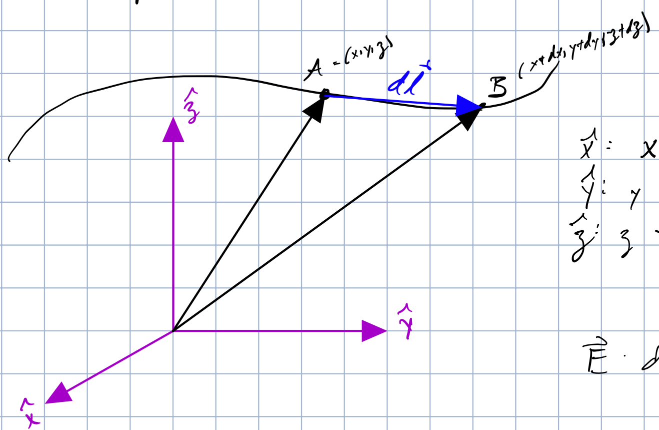




$\vec{R} = \vec{r} - \vec{r}' = \vec{\mathscr{r}}$ (Griffiths)

$$\hat{R} = \frac{(x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} = \frac{\vec{R}}{|\vec{R}|}$$

infinitesimal displacement



$\hat{x}$: $x \to x+dx$
$\hat{y}$: $y \to y+dy$
$\hat{z}$: $z \to z+dz$

$$\vec{E} \cdot d\vec{l} = (E_x\hat{x} + E_y\hat{y} + E_z\hat{z}) \cdot (dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z})$$
$$= E_x dx + E_y dy + E_z dz$$

Position vector

Cartesian $\quad \vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

Cylindrical $\quad \vec{r} = s\cdot\hat{s} + z\cdot\hat{z}$
$$= s\cdot\cos\phi\cdot\hat{x} + s\cdot\sin\phi\,\hat{y} + z\cdot\hat{z}$$

$\hat{s} \perp \hat{\phi} \perp \hat{r}$

$\hat{s} = \cos\phi\cdot\hat{z} + \sin\phi\,\hat{y}$

$\hat{\phi}$ = rotation of $\hat{s}$ by 90° in ϕ
$$= \cos(\phi+90°)\,\hat{x} + \sin(\phi+90°)\,\hat{y}$$
$$= -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$

Spherical

$\vec{r} = r\sin\theta\cos\phi\,\hat{x} + r\sin\theta\sin\phi\,\hat{y} + r\cos\theta\,\hat{z}$

$\hat{\theta}$ = rotation of $\hat{r}$ by $90°$ in θ

$= \hat{r}(\theta \to \theta + 90°)$ — change each θ to $\theta + 90°$

$= \cos\theta\cdot\cos\phi\,\hat{x} + r\cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}$

$\hat{\phi}$ = rotation of $\hat{r}$ by $90°$ in ϕ

$= \hat{r}(\phi \to \phi + 90°)$ — change each ϕ to $\phi + 90°$

$= -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$

$\hat{r} = \hat{r}(\theta, \phi)$
$\hat{\theta} = \hat{\theta}(\theta, \phi)$
$\hat{\phi} = \hat{\phi}(\phi)$

More Displacement & Volume

Cartesian:

$d\vec{l} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$
$d\tau = dx\cdot dy\cdot dz$

Cylindrical:

$d\vec{l} = dl_s\cdot\hat{s} + dl_\phi\cdot\hat{\phi} + dl_z\,\hat{z}$ $\qquad dl_s = ds \qquad dl_z = dz$
$\qquad dl_\phi = s\,d\phi$

$d\vec{l} = ds\,\hat{s} + s\cdot d\phi\,\hat{\phi} + dz\,\hat{z}$

$d\tau = s\cdot ds\cdot d\phi\cdot dz$

Spherical:

$d\vec{l} = dl_r\,\hat{r} + dl_\theta\,\hat{\theta} + dl_\phi\cdot\hat{\phi}$

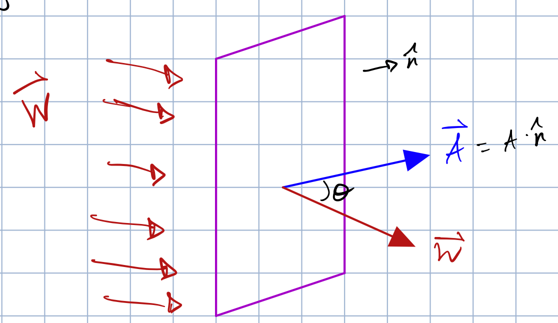
$d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$

$d\tau = dr\cdot r\,d\theta\cdot r\sin\theta\,d\phi$

Flux & Circulation

Flux describes effects that appear to pass thru surface

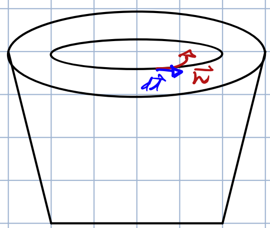
Wind thru window



flux $= \vec{W}\cdot\vec{A}$
$= W\cdot A\cos\theta$
$= A\cdot \underbrace{W\cos\theta}_{\text{component normal to wind}}$

Circulation describes net rotational motion of field along edge of closed loop

water stirred in glass



Gradient $\quad \vec{\nabla} \cdot T$ $\quad\quad T$ is scalar

1D : slope
2D/3D : max slope

$\vec{\nabla} T$ points in max. increase of $f = T$

$|\vec{\nabla} T|$ gives slope along this max. direction

Del Operator $\quad$ (gradient if on scalar)

Cartesian: $\quad \vec{\nabla} = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z}$

Cylindrical : $\quad \vec{\nabla} = \hat{s} \frac{\partial}{\partial s} + \frac{1}{s} \hat{\phi} \frac{\partial}{\partial \phi} + \hat{z} \frac{\partial}{\partial z}$

Spherical : $\quad \vec{\nabla} = \hat{r} \frac{\partial}{\partial r} + \frac{1}{r} \hat{\theta} \frac{\partial}{\partial \theta} + \frac{1}{r \sin\theta} \hat{\phi} \frac{\partial}{\partial \phi}$

Divergence $\quad \vec{\nabla} \cdot \vec{A}$

acts via dot product

"spreading out" >0 or "sinking" <0 or same $=0$

Cartesian: $\quad \vec{\nabla} \cdot \vec{A} = \left(\hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} \right) \cdot \left(A_x \hat{x} + A_y \hat{y} + A_z \hat{z} \right)$

$$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Cylindrical: $\quad \vec{\nabla} \cdot \vec{A} = \left(\hat{s} \frac{\partial}{\partial s} + \frac{1}{s} \hat{\phi} \frac{\partial}{\partial \phi} + \hat{z} \frac{\partial}{\partial z} \right) \cdot \left(A_s \hat{s} + A_\phi \hat{\phi} + A_z \hat{z} \right)$

⋮ work

$$= \frac{1}{s} \frac{\partial}{\partial s}(s \cdot A_s) + \frac{1}{s} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

Spherical: $\vec{\nabla} \cdot \vec{A} = \left(\hat{r} \frac{\partial}{\partial r} + \frac{1}{r} \hat{\theta} \frac{\partial}{\partial \theta} + \frac{1}{r\sin\theta} \hat{\phi} \frac{\partial}{\partial \phi}\right) \cdot \left(A_r \cdot \hat{r} + A_\phi \cdot \hat{\phi} + A_\theta \cdot \hat{\theta}\right)$

$$= \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta} \frac{\partial}{\partial \theta}(\sin\theta \cdot A_\theta) + \frac{1}{r\sin\theta} \frac{\partial A_\phi}{\partial \phi}$$

Curl $\vec{\nabla} \times \vec{A}$

act w/cross product

Second Derivatives

Divergence of Gradient

$\vec{\nabla} \cdot (\vec{\nabla} T) \equiv \vec{\nabla}^2 T$ Laplacian

Curl of a Gradient

$\vec{\nabla} \times (\vec{\nabla} \cdot T) = \ldots = 0$

Gradient of Divergence

$\vec{\nabla}(\vec{\nabla} \cdot \vec{T}) = 0$

Divergence of Curl

$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$

Curl of curl

$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 \vec{A}$