Vector Calculus stuff (1.1 & 1.2)

consider space

point $P(x,y,z)$ $\vec{E} \neq 0$ $\vec{B} \neq 0$

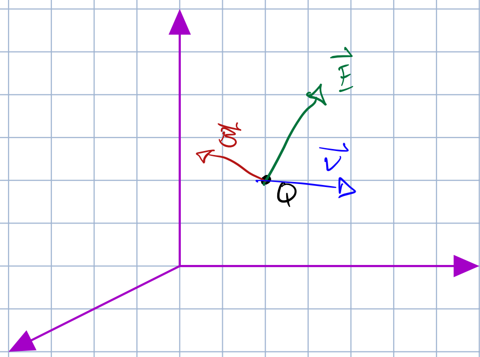
$\vec{E}(x,y,z)$ & $\vec{B}(x,y,z)$ generated by charges & current somewhere else

put test charge Q

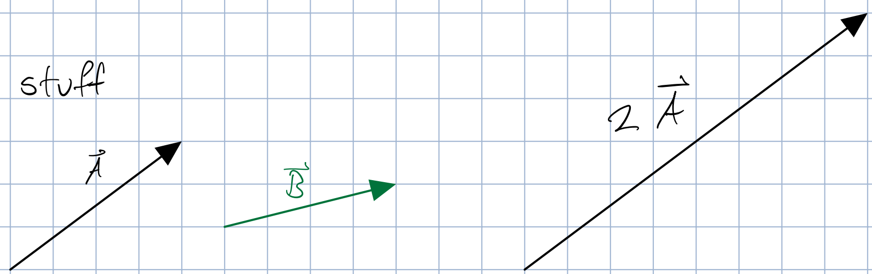
$\vec{F} = Q \cdot \vec{E} + Q \vec{v} \times \vec{B}$

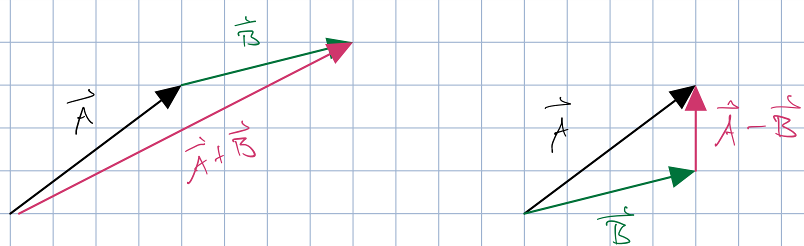
$\vec{F}, \vec{E}, \vec{v}, \vec{B}$ → vectors

Q → scalar (no direction)



Vector stuff

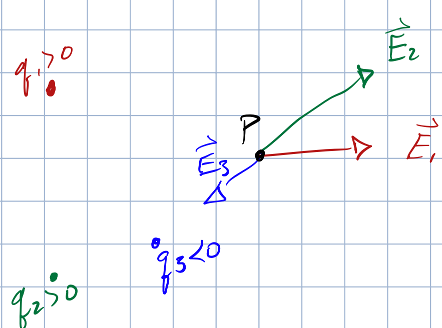




Associative $(A+B)+C = A+(B+C)$

Commutative $A+(B+C) = (B+C)+A$

Distributive $a(B+C) = aB + aC$



total $\vec{E}$ @ point P: $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \vec{E}_{Tot}$

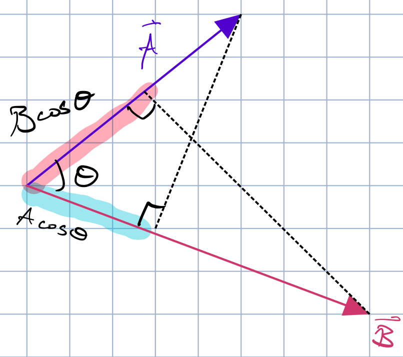
place test charge. force? → $F_{Tot} = Q\vec{E}_{Tot}$

sum indv. contributions

Dot Product

apply 2 vectors

$\vec{A} \cdot \vec{B}$

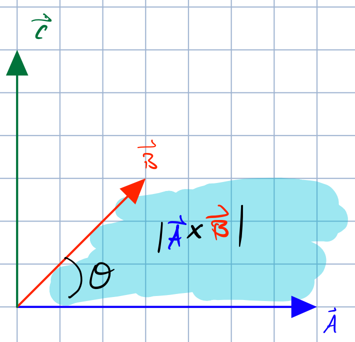


$\theta \in [0, \pi]$

angle between $\vec{A}$ & $\vec{B}$

$\vec{C} \cdot \vec{C} = C \cdot C \cos(0) = C^2$

Cross Product



$\vec{A} \times \vec{B} = \vec{C}$

magnitude $= |\vec{C}| = |\vec{A}| \cdot |\vec{B}| \cdot \sin\theta$

$\theta \in [0, \pi]$

direction : right hand rule

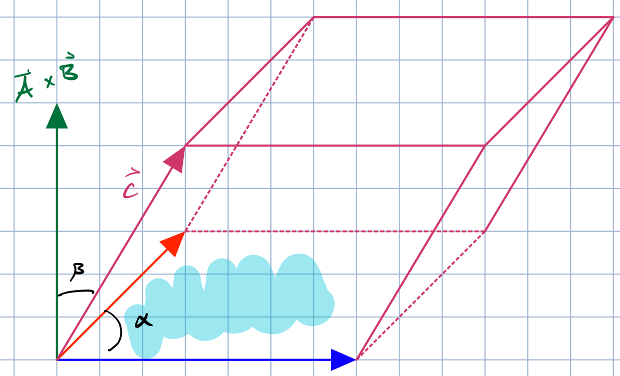
$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A}) = -\vec{B} \times \vec{A}$

$\vec{A} \times \vec{A} = 0$

$\vec{A} \times \vec{C}$

$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

Triple Product



$(\vec{A} \times \vec{B}) \cdot \vec{C}$ = scalar

$(\vec{A} \times \vec{B}) \cdot \vec{C} = |a \times b| \cdot |c| \cdot \cos\beta$

$= |a| \cdot |b| \cdot |c| \cdot \sin\alpha \cdot \cos\beta$

$=$ Volume of parallelepiped

$(\vec{A} \times \vec{B}) \times \vec{C} \neq \vec{A} \times (\vec{B} \times \vec{C})$ (in general)

$\vec{A} \times (\vec{B} \times \vec{C}) = B(A \cdot C) - C(A \cdot B)$

$(\vec{A} \times \vec{B}) \times \vec{C} = -\vec{C} \times (\vec{A} \times \vec{B})$

$= -(A \cdot (C \cdot B) - B \cdot (C \cdot A))$

$= B \cdot (A \cdot C) - A \cdot (B \cdot C)$

Coordinate System

cartesian $x\ y\ z$

cylindrical $s\ \phi\ z$

spherical $r\ \phi\ \theta$

$$\vec{A} \times \vec{B} = \left|\begin{array}{ccc|cc} x & y & z & x & y \\ A_x & A_y & A_z & A_x & B_y \\ B_x & B_y & B_z & B_x & A_y \end{array}\right| = \hat{x}\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} + \hat{y}\begin{vmatrix} A_z & A_x \\ B_z & B_x \end{vmatrix} + \hat{z}\begin{vmatrix} A_x & B_y \\ B_x & A_y \end{vmatrix}$$