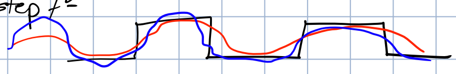
Seperation of Variables

completeness & orthogonality

Fourier series $\quad f(x) = a_0 + \sum \left(a_n \cos(\alpha n x) + b_n \cdot \sin(\alpha n x)\right)$

applying to step f



more terms $\rightarrow$ closer to f

$f_n(x) \quad , \; n = 1, 2, \ldots, \infty$

$\int_a^b f_n(x) \cdot f_m(x)\, dx = \delta_{mn} \quad \longrightarrow$ orthogonality

$f(x) = \sum f_n(x) \quad \longrightarrow$ completeness

$\left\{ \underset{f_0}{\frac{1}{\sqrt{2\pi}}},\; \underset{f_1}{\frac{1}{\sqrt{\pi}} \cos(x)},\; \underset{f_2}{\frac{1}{\sqrt{\pi}} \cos(2x)}, \ldots \right\}$

Legendre Polynomials

$$P_l(x) = \frac{1}{2^l \cdot l!} \cdot \frac{d^l}{dx^l}\left[(x^2-1)^l\right]$$

$$\int_{-1}^{1} P_n(x) P_m(x)\, dx = \frac{2}{2l+1} \delta_{mn}$$

$$\nabla^2 V(x,y,z) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$V(x,y,z) \equiv f(x) g(y) h(z)$$

$$\frac{\partial V}{\partial x} = g(y) h(z) \cdot \frac{df}{dx}$$

$$\frac{\partial^2 V}{\partial x^2} = g(y) h(z) \cdot \frac{d^2 f}{dx^2} = \frac{f(x) g(y) h(z)}{f(x)} \cdot \frac{d^2 f}{dx^2} = \frac{1}{f} \cdot \frac{d^2 f}{dx^2} V$$

$$\frac{\partial^2 V}{\partial y^2} = \frac{1}{g} \frac{d^2 g}{dy^2} \cdot V$$

$$\frac{\partial^2 V}{\partial z^2} = \frac{1}{h} \frac{d^2 h}{dz^2} \cdot V$$

Laplacian $\rightarrow \quad \frac{1}{f} \frac{d^2 f}{dx^2} + \frac{1}{g} \frac{d^2 g}{dy^2} + \frac{1}{h} \frac{d^2 h}{dz^2} = 0$

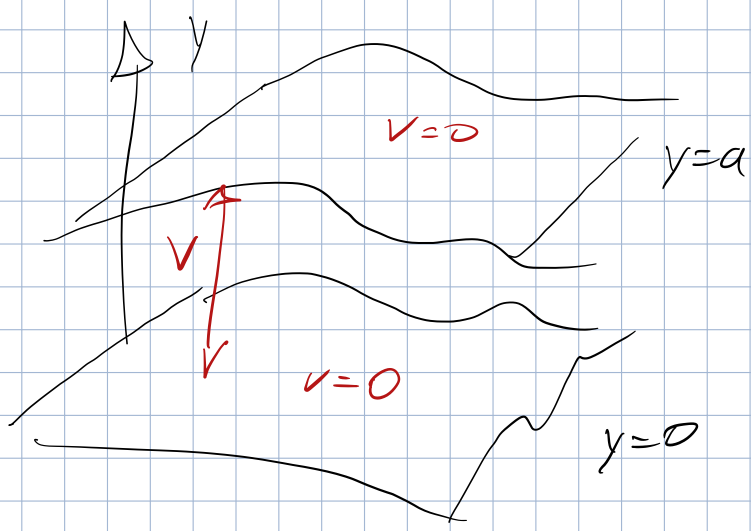
$$\rightarrow \begin{cases} \frac{1}{f} \frac{d^2 f}{dx^2} = \alpha^2 \\ \frac{1}{g} \frac{d^2 g}{dy^2} = \beta^2 \\ \frac{1}{h} \frac{d^2 h}{dz^2} = \underbrace{-\gamma^2}_{\rightarrow (i\gamma)^2} \end{cases} \xrightarrow{\text{sol'n}} \begin{cases} f = a_1 e^{\alpha x} + a_2 e^{-\alpha x} \\ g = b_1 e^{\beta x} + b_2 e^{-\beta x} \\ h = c_1 e^{i\gamma x} + c_2 e^{-i\gamma x} \end{cases}$$

boundary condition $\alpha^2 + \beta^2 = \gamma^2$

$$V(x,y,z) = \sum \left(a_1^{\alpha} e^{\alpha x} + a_2^{\alpha} e^{-\alpha x}\right)\left(b_1^{\beta} e^{\beta x} + b_2 e^{-\beta x}\right)\left(c_1 e^{\gamma x} + c_2 e^{-\gamma x}\right)$$

$V \to 0$ as $x \to \infty$ $\quad V \sim e^{-\alpha x}$

$V \to 0$ as $z \to \infty$ $\quad \left(\frac{1}{h}\frac{d^2h}{dz^2} > 0\right)!$ $\quad h = c_1 e^{\gamma z} + c_2 e^{-\gamma z} \longrightarrow c_1 = 0$



$$V(x,y,z) = V(x,y)$$

$$\underbrace{\frac{\partial^2 V}{\partial^2 x}}_{\alpha^2} + \underbrace{\frac{\partial^2 V}{\partial y^2}}_{\beta^2}$$