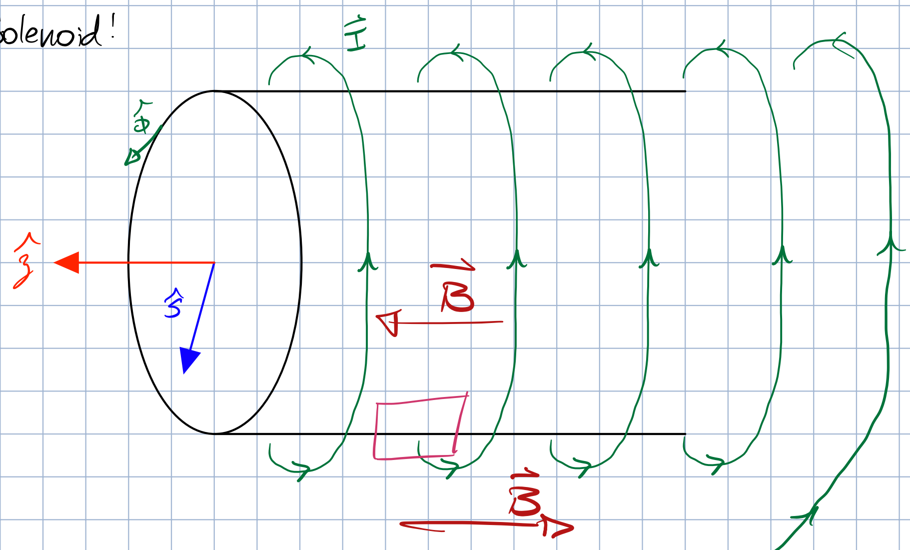
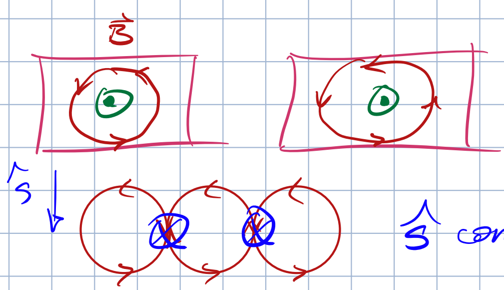
Example: Solenoid!



$$\vec{B} = B_s \hat{s} + B_\phi \cdot \hat{\phi} + B_z \cdot \hat{z}$$

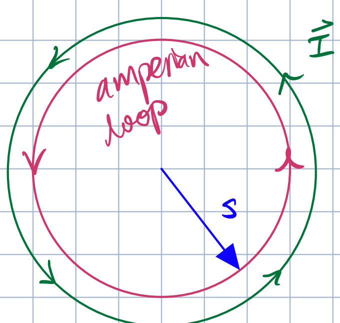
$B_s = ???$



$\hat{s}$ components cancel

$B_s = 0$

$B_\phi = ???$



$$\oint \vec{B} \cdot d\vec{l} = \oint \vec{B} \cdot dl\, \hat{\phi}$$

$$= \oint B_\phi \cdot dl$$

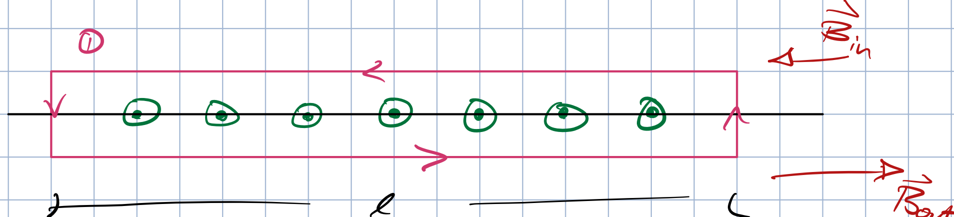
ϕ symmetry

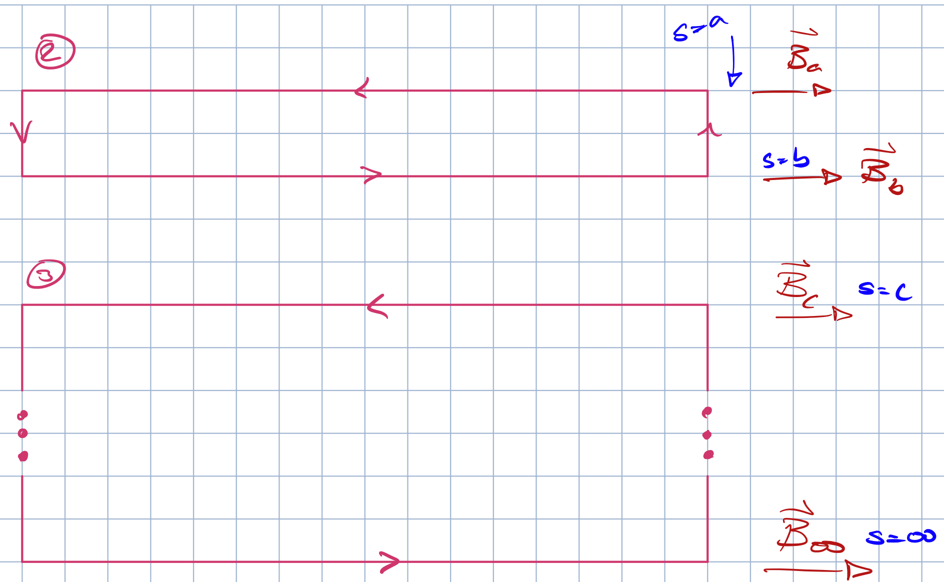
$$= B_\phi \oint dl$$

$$= B_\phi\, 2\pi s = \mu_0 \underbrace{I_{encl.}}_{\to 0}$$

$\longrightarrow B_\phi = 0$

$B_z = ???$





① $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} = B_{in}\cdot l + B_{out}\cdot l = \mu_0 n l I$ $n = \#$ of wires/length

$$\star\ B_{in} + B_{out} = \mu_0 n I$$

② $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} = B_b\cdot l - B_a l \overset{*}{=} 0$ $I_{enc} = 0$

$$\rightarrow B_a = B_b$$

$\rightarrow B_{out}$ is same everywhere outside

③ $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} = -B_c\cdot l \overset{*}{=} 0$ $I_{encl} = 0$

$$\rightarrow B_c = 0$$

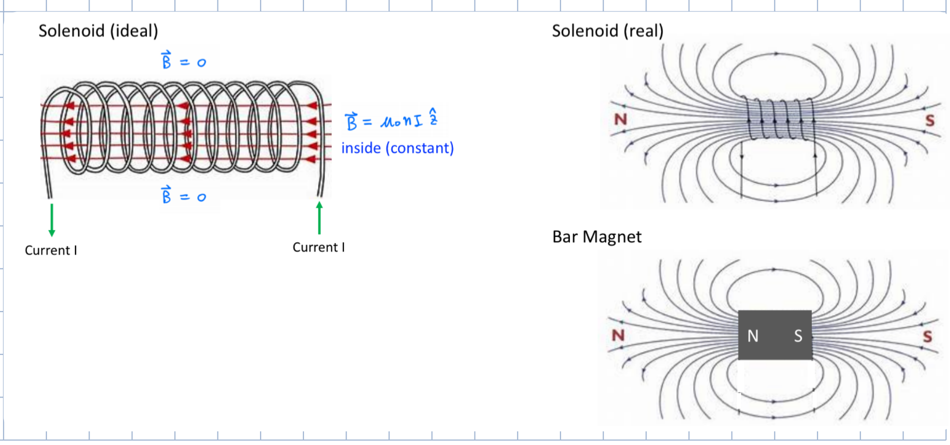
$$B_c = B_b = B_a = B_{out}$$

$$\star \rightarrow B_{in} = B = \mu_0 n I$$

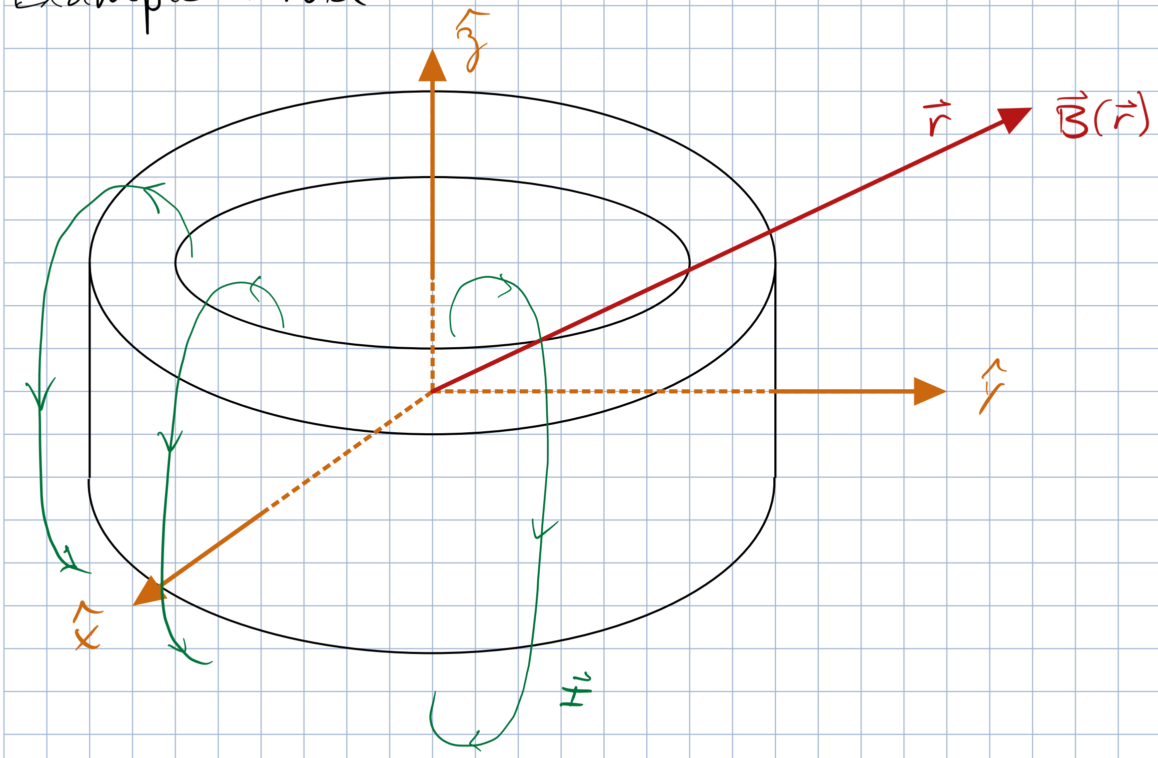
outside: $\vec{B} = 0$

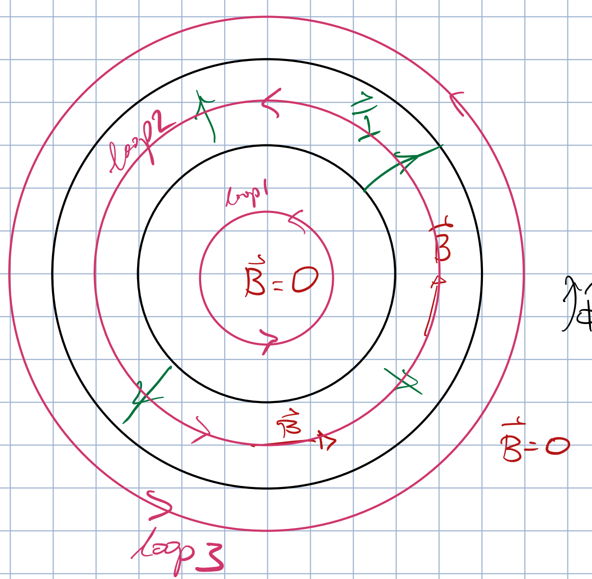
inside: $\vec{B} = \mu_0 n I \hat{z}$

(constant)



Example: Toroid





N total turns

$\vec{B} = B\,\hat{\phi}$

$B_s = 0$
$B_z = 0$

due to ϕ symmetry, B isn't f^n of ϕ

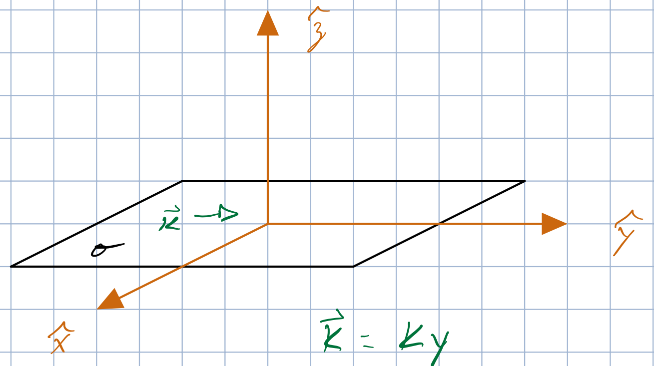
Amperian loops

$d\vec{l} = s \cdot d\phi \cdot \hat{\phi}$
$\vec{B} = B\,\hat{\phi}$

for all 3 loops: $\oint \vec{B} \cdot d\vec{l} = 2\pi s B = \mu_0 I_{encl}$

loop 1: $I_{encl} = 0 \longrightarrow B_1 = 0$ $\qquad \vec{B}_1 = 0$
loop 2: $I_{encl} = N \cdot I \longrightarrow B_2 = \frac{\mu_0 I N}{2\pi} \cdot \frac{1}{s}$ $\qquad \vec{B}_2 = \frac{\mu_0 I N}{2\pi} \cdot \frac{1}{s}\,\hat{\phi}$
loop 3: $I_{encl} = 0 \longrightarrow B_3 = 0$ $\qquad \vec{B}_3 = 0$

Example: Infinite Surface



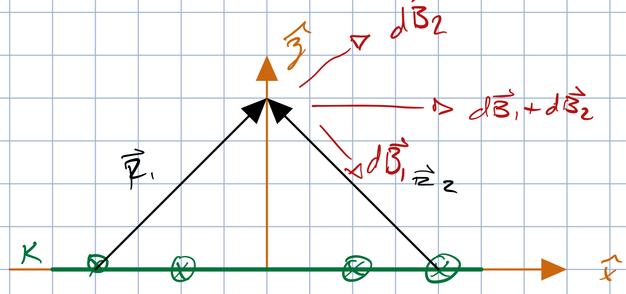
$\vec{K} = Ky$

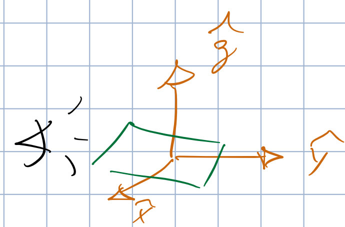
$$\vec{B} = B_x \cdot \hat{x} + B_y \cdot \hat{y} + B_z \cdot \hat{z}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{K} \times \hat{\mathscr{r}}}{\mathscr{r}^2} da' = \frac{\mu_0}{4\pi} \vec{K} \times \int \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} da'$$

$\vec{B}$ & $\vec{I}$ are $\perp$ $\rightarrow$ $B \perp K$ $\rightarrow$ $B_y = 0$

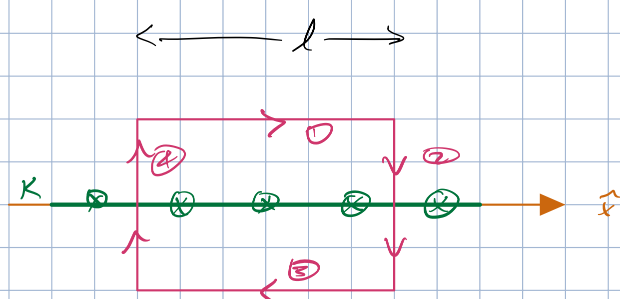
$$\vec{B} = B_x \, \hat{x} + B_z \cdot \hat{z}$$





B_z components cancel: $B_z = 0$

$\vec{B} = B \hat{x}$



$$d\vec{l}_1 = dx \cdot \hat{x} = dx \, \hat{x}$$
$$d\vec{l}_2 = dz \cdot (-\hat{z}) = -dz \, \hat{z}$$
$$d\vec{l}_3 = dx \cdot (-\hat{x}) = -dx \, \hat{x}$$
$$d\vec{l}_4 = dz \cdot \hat{z} = dz \, \hat{z}$$

$\vec{B} = B \hat{x}$

$\oint \vec{B} \cdot d\vec{l} \rightarrow \underbrace{\vec{B} \cdot d\vec{l}_2}_{\hat{x} \cdot -\hat{z}} = \underbrace{\vec{B} \cdot d\vec{l}_4}_{\hat{x} \cdot \hat{z}} = 0$

$$\oint \vec{B}\cdot d\vec{l} = B \oint \hat{x}\cdot d\vec{l} = B_1 \int \hat{x}\,dx\cdot\hat{x} - B_3 \int \hat{x}\,dx\,\hat{x}$$

$$= B_1 \int dx - B_3 \int dx = B_1 \cdot l - B_3 \cdot l = l(B_1 - B_3)$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} = \mu_0 \cdot K l$$

$$\rightarrow \quad l(B_1 - B_3) = \mu_0 K l$$

$$B_1 - B_3 = \mu_0 K$$

by symmetry, $|B_1| = |B_3| \rightarrow B_3 = -B_1$

$$\rightarrow 2B_1 = \mu_0 K \rightarrow B_1 = \tfrac{1}{2}\mu_0 K$$