Electrostatics

constant $\vec{E}$
charges @ rest
no $\vec{B}$

Electric force on charges

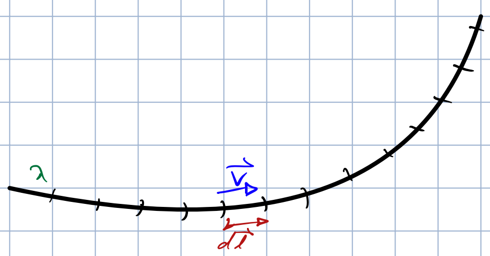
Magnetostatics

constant $\vec{B}$
moving charges, constant velocity, Onet charge
do not produce $\vec{E}$

magnetic force on moving charges/current

Lorentz Force: $\vec{F} = q\vec{E} + q\,\vec{v}\times\vec{B}$

current $\equiv$ charge per unit time moving @ 1 point



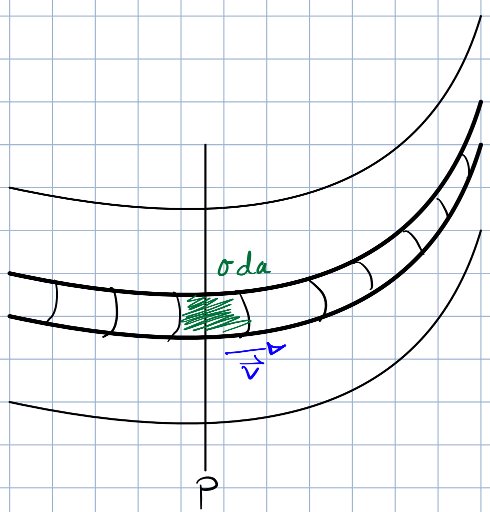
current $\equiv$ $\underbrace{\vec{I} = \lambda\vec{v}}$

charge λdl is moving w/ $\vec{v}$

force on this charge is $\vec{F} = q\vec{v}\times\vec{B}$

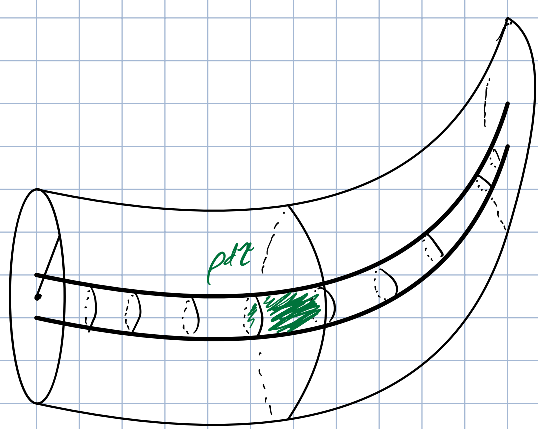
$$\vec{F} = \int \vec{v}\times\vec{B}\;\lambda\, d\vec{l} = \int \underbrace{\lambda\vec{v}}\times\vec{B}\cdot d\vec{l}$$

$$= \int \vec{I}\times\vec{B}\, d\vec{l} = \int \left(\vec{I}\,d\vec{l}\times\vec{B}\right)$$



current per width $\equiv$ $\vec{K} = \sigma\vec{v}$

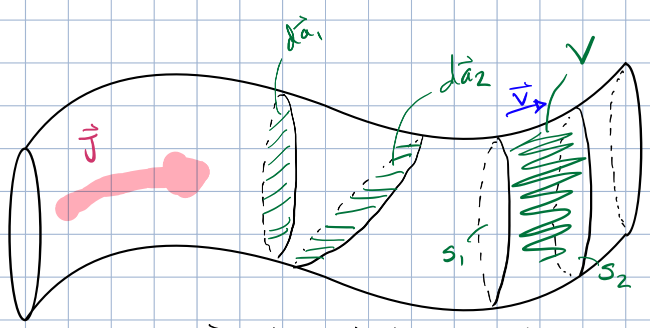
$$\vec{F} = \int \vec{v}\times\vec{B}\;\sigma d\vec{a} = \int \sigma\vec{v}\times\vec{B}\, d\vec{a} = \int (\vec{K}\times\vec{B})\, d\vec{a}$$



current per area $\equiv$ $\vec{J} = \rho\vec{v}$

$$\vec{F} = \int \vec{v}\times\vec{B}\;\rho d\tau = \int \rho\vec{v}\times\vec{B}\, d\tau = \int (\vec{J}\times\vec{B})\, d\tau$$

$\vec{I}$ = current $\qquad$ $\vec{K}$ = surface current density $\qquad$ $\vec{J}$ = volume current density



$\vec{J}\cdot d\vec{a}_1 = \vec{J}\cdot d\vec{a}_2 = J\,da_\perp$ similar to flux

$$\oint_S \vec{J}\cdot d\vec{a} = \int_V (\vec{\nabla}\cdot\vec{J})\,d\tau \qquad \text{Divergence Th}^{\underline{m}}$$

① Steady current: $Q_{in} = Q_{out}$

$\vec{J}$ does not change w/ time

$$\oint \vec{J}\cdot d\vec{a} = \underbrace{\int_{S_1}\vec{J}\cdot d\vec{a} + \int_{S_2}\vec{J}\cdot d\vec{a}}_{\text{net "charge flux"}=0} + \underbrace{\int_{sides}\vec{J}\cdot d\vec{a}}_{=0} = 0 = \int_V (\vec{\nabla}\cdot\vec{J})\,d\tau$$

$$\Rightarrow \vec{\nabla}\cdot\vec{J} = 0$$

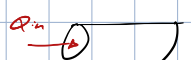
② Non steady current

Say $Q_{in} < Q_{out}$

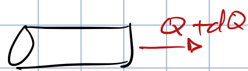
outside volume: $\underbrace{\oint \vec{J}\cdot d\vec{a}}_{\text{charge flux / time}} > 0$ conserved charge

inside volume: $\oint \vec{J}\cdot d\vec{a} < 0$

@ $t = t_1$



@ $t = t_1 + dt$



$dQ > 0$

$$-\frac{dQ}{dt} = -\frac{d}{dt}\int_V \rho\,d\tau = -\int_V \frac{\partial\rho}{\partial t}\,d\tau$$

$$\oint \vec{J}\cdot d\vec{a} = \int_V (\vec{\nabla}\cdot\vec{J})\,d\tau = -\int \frac{\partial\rho}{\partial t}\,d\tau$$

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

for steady state $\frac{\partial\rho}{\partial t} = 0$

electrons are the current carriers

Steady current

if 1 electron moves out of way, another e^- moves into replace instantaneously

no charge accumulation

[charge] = Coulomb

[current] = Coulomb · sec^{-1} = Ampere

current $I = \vec{J} \cdot \vec{A}$