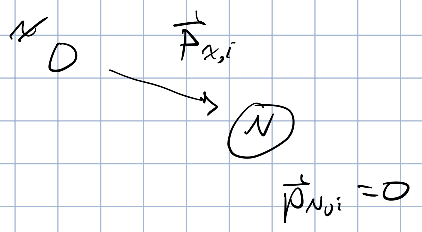
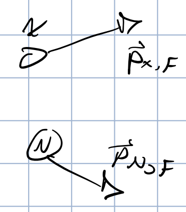
10 mins late :(

Rate ~ R ~ $n_\chi \sigma v$ per nucleus

local density of DM; x-section $\chi N \to \frac{\chi N}{E_R}$; relative velocity

$\Phi = n_\chi \cdot v = \frac{1}{vol} \cdot \frac{length}{time} = \frac{1}{area \cdot time}$





$E_R = \frac{1}{2M_N} \cdot |\vec{p}_{NF}|^2$

goal: $\frac{dR}{dE_R}$ $E_R > E_{thresh} \sim keV$

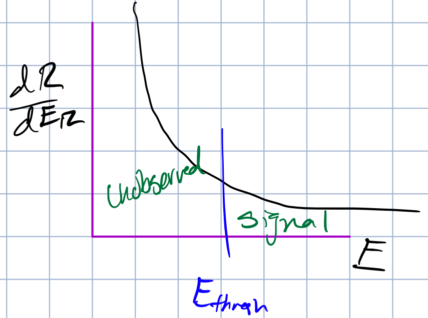
$$\frac{dR}{dE_R} = N_T \cdot n_\chi^{\odot} \cdot \int d^3v \, \underbrace{f(\vec{v})}_{\text{velocity distribution}} \frac{d\sigma}{dE_R} \cdot |\vec{v}|$$

N_T: total # of detector nuclei

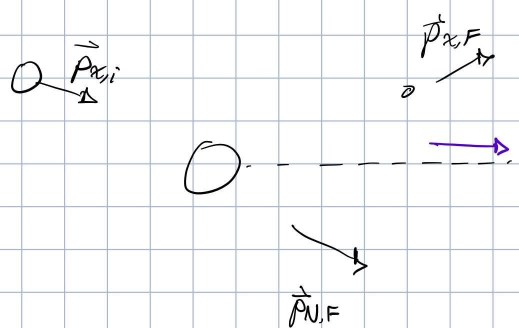
$n_\chi^{\odot}$: local DM # density, $n_\chi^{\odot} = \frac{\rho_\chi^{\odot}}{m_\chi}$ known

$\frac{d\sigma}{dE_R}$: differential x-section from QFT

limits of integration



fix E_R. what's the min "v" for E_R?



most efficient way to share KE w/ nucleus is a $\vec{p}_{\chi,F}$ parallel

non relativistic energy conservation

$$\frac{|\vec{p}_{\chi,i}|^2}{2m_\chi} = \frac{|\vec{p}_{\chi,F}|^2}{2m_\chi} + \frac{|\vec{p}_{N,F}|^2}{2m_N}$$

momentum conservation: $\vec{p}_{\chi,i} + \vec{p}_{\chi,F} = \vec{p}_{N,F} \to \vec{p}_{\chi,F} = \vec{p}_{N,F} - \vec{p}_{\chi,i}$

$$\to \frac{|\vec{p}_{\chi,i}|^2}{2m_\chi} = \frac{|\vec{p}_{N,F} - \vec{p}_{\chi,i}|^2}{2m_\chi} + \frac{|\vec{p}_{N,F}|^2}{2m_N}$$

$$\frac{\cancel{|\vec{p}_{\chi,i}|^2}}{2m_\chi} = \frac{|\vec{p}_{N,F}|^2 + \cancel{|\vec{p}_{\chi,i}|^2} - 2\,\vec{p}_{\chi,i} \cdot \vec{p}_{N,F}}{2m_\chi} + \frac{|\vec{p}_{N,F}|^2}{2m_N}$$

$$\frac{\vec{p}_{\chi i}\cdot\vec{p}_{N,F}}{m_\chi} = \frac{|\vec{p}_{NF}|^2}{2}\left(\frac{1}{m_\chi}+\frac{1}{m_N}\right) \qquad \mu \equiv \frac{m_\chi \cdot m_N}{m_\chi + m_N}$$

$$\longrightarrow \quad \frac{\vec{p}_{\chi i}\cdot\vec{p}_{N,F}}{m_\chi} = \frac{|\vec{p}_{N,F}|^2}{2\mu}$$

for min KE $\quad \vec{p}_{\chi i}\cdot\vec{p}_{N,F} = |\vec{p}_{\chi i}|\cdot|\vec{p}_{N,F}| \qquad \cos\theta = 1$

in collinear limit: $\quad |\vec{p}_{\chi i}| = m_\chi \cdot V_{min}(E_R)$

$$\longrightarrow \quad \frac{|\vec{p}_{\chi i}|\cdot\cancel{|\vec{p}_{N,F}|}}{m_\chi} = \frac{|\vec{p}_{N,F}|^{\cancel{2}}}{2\mu}$$

$$\longrightarrow \quad \frac{m_\chi \cdot V_{min}}{m_\chi} = \frac{|\vec{p}_{N,F}|}{2\mu}$$

$$V_{min}(E_R) = \frac{|\vec{p}_N|}{2\mu} = \frac{\sqrt{2 m_N \cdot E_R}}{2\mu}$$

$V < V_{escape} \approx 550\ \mathrm{km/s}$

local escape velocity . galaxy rest frame

$\int_{V_{min}}^{V_{esc}}$

Velocity distribution

$\int f(v)\, dv^3 = 1$

if DM is pure self-gravitating gas

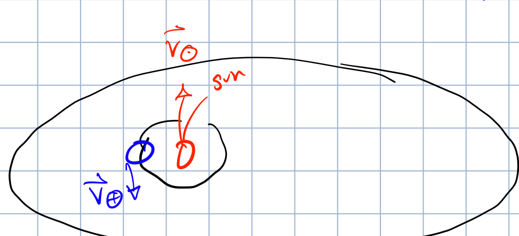
step f^n: $+1$ if $v_{esc} > |\vec{v}|$; 0 if $v_{esc} < |\vec{v}|$

in galaxy frame: $\quad f_{gal}(\vec{v}) = N \exp\left[-\frac{|\vec{v}|^2}{v_0^2}\right]\Theta(v_{esc} - |\vec{v}|)$

normalization
$N = \left(\int d^3v\, f(\vec{v})\right)^{-1}$

$v_0 = 220\ \mathrm{km/s}$
local dispersion for DM
velocity spread

Boost into lab frame:



$$\vec{V}_e(t) = \vec{V}_0 + \vec{V}_\oplus(t)$$

$|\vec{V}_\oplus| \simeq 30\,km/s$

$\vec{V}_\oplus \propto \cos(\omega t) \qquad \omega = 1yr$

↳ $\vec{V}_\oplus$ can be aligned w/ $\vec{V}_0$

DAMA experiment

$|\vec{V}_0| \simeq 230\,km/s$

$$f_{lab}(\vec{v}) = f_{gal}(\vec{v} + \vec{V}_e(t))$$

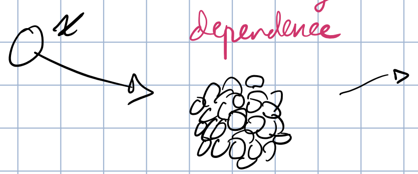
Differential Cross section $\frac{d\sigma}{dE_R}$

if elastic scatter & spin indpt

$$\rightarrow \quad \frac{d\sigma}{dE_R} = \underbrace{\sigma_0}_{\text{total single nucleon x-sec}} \cdot \frac{m_N}{2\mu^2} \cdot \frac{1}{V^2} \underbrace{F(E_R)^2}_{\text{form factor}}$$

total single nucleon x-sec
includes $A \sim Z$ chemical dependence

form factor
nuclear physics (nucleus is composite object)



$F(0) \approx 1$

$F(E_R \gg MeV) \approx e^{-E_R/m_N}$

in QM $\rightarrow \sigma \propto |\Sigma A|^2$

$$\frac{dR}{dE_R} = \frac{N_T \cdot \rho_\chi^\odot}{m_\chi} \cdot \frac{\sigma_0 m_N}{2\mu^2} \cdot F(E_R)^2 \cdot \int_{V > V_{min}(E_R)} d^3v \cdot \frac{f(\vec{v})}{|\vec{v}|}$$

$$\text{define} \quad \eta(E_R) \equiv \int_{V > V_{min}(E_R)}^{V_{esc}} \frac{d^3v}{|\vec{v}|} f(\vec{v})$$

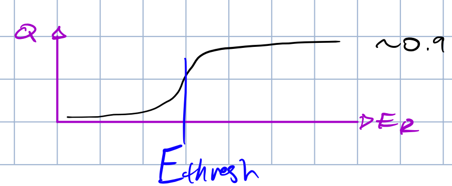
inverse measure speed

$$\frac{dR}{dE_R} = \underbrace{(N_T \cdot m_N)}_{\text{mass of detector } M_{Det}} \cdot \frac{\sigma_0}{2\mu} \cdot \frac{\rho_\chi^\odot}{m_\chi} \cdot F(E_R)^2 \cdot \underbrace{\eta(E_R)}_{\text{inverse mean speed}}$$

mass of detector M_{Det}

↳ inverse mean speed

Efficiency fn $Q(E_R)$



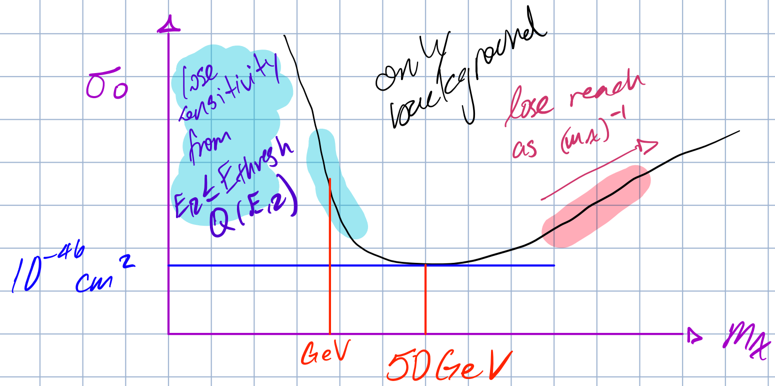
arctan shape

Run experiment for Δt

cuts off @ high E_R cuts off @ low E_R

$$\frac{dN_{signal}}{dE_R} = \Delta t \cdot \frac{dR}{dE_R} = \underbrace{\Delta t \cdot M_{det}}_{\text{"exposure" kg}\cdot\text{day}} \cdot \frac{\sigma_0}{2\mu^2} \cdot \frac{\rho_\chi^0}{m_\chi} \cdot F(E_R)^2 \cdot \eta(E_R)\, Q(E_R)$$

$$\int_{v>v_{min}(E_R)}^{v_{esc}}$$



mass dependence