last time: $\delta(r,t) \equiv \dfrac{\rho_m(r,t) - \bar{\rho}_m(t)}{\bar{\rho}_m(t)}$

Correlation f^{n}: $C(\vec{R}) = \left\langle \sum_i \delta(\vec{r}_i)\, \delta(\vec{r}_i + \vec{R}) \right\rangle_{|\vec{R}| = \text{const}}$

continuum: $C(\vec{R}) = \int d^3r\, \delta(\vec{r})\, \delta(\vec{r} + \vec{R})$

Fourier Transform: $\delta(\vec{r}) = \int \dfrac{d^3k}{(2\pi)^3}\, e^{-i\vec{k}\cdot\vec{r}}\, \underbrace{\tilde{\delta}_k(t)}_{f^n \text{ of } k = \frac{2\pi}{\lambda}}$

⋮

$C(\vec{R}) = \int \dfrac{d^3k}{(2\pi)^3}\, e^{-i\vec{k}\cdot\vec{R}}\, |\tilde{\delta}_k|^2 \rightarrow$ power spectrum $P(k)$

solve for via Equation of Motion: $\ddot{\delta} + 2H\dot{\delta} - \dfrac{c_s^2}{a^2} \cdot \nabla^2 \delta - 4\pi G \bar{\rho}_m \delta = 0$

solve using F.T.

$\rightarrow \quad \ddot{\delta}_k + 2H\dot{\delta}_k + \left(\dfrac{c_s^2}{a^2} k^2 - 4\pi G \bar{\rho}_m\right) \delta_k = 0$

ignoring expansion $(H = 0)$ & $a = 1$

$\rightarrow \quad \ddot{\delta}_k + \left(\dfrac{c_s^2}{a^2} k^2 - 4\pi G \bar{\rho}_m\right) \delta_k = 0$

look for sol^{n}s of type $\delta_k = A e^{i\omega k t}$

$\longrightarrow \quad \omega^2 = \underbrace{\dfrac{c_s^2}{a^2} k^2 - 4\pi G \bar{\rho}_m}$

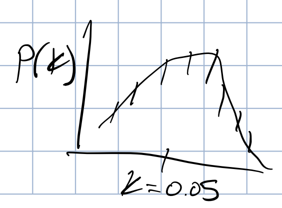
critical side when RHS $= 0$

balanced @ $\left(\dfrac{2\pi}{\lambda_J}\right)^2 c_s^2 = 4\pi G \bar{\rho}_m$

def^{n} of Jeans length $\rightarrow \lambda_J \equiv c_s \left(\dfrac{\pi}{G\bar{\rho}_m}\right)^{1/2}$

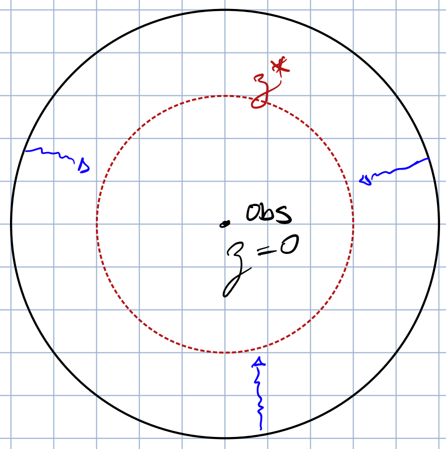
find Power Spectrum: $P(k) = |\delta_k|^2$

MASS



$[Mpc]^{-1}$

CMB



$z \equiv$ redshift

$$\frac{a_0}{a(t)} = 1+z$$

different $P(k)$ @ each redshift

$P(k,t)$ $\quad$ $P(k, z^*)$ → 3D observable

CMB → 2D $\quad$ released from surface @ $z = 1100$ right after γ-decoupling

$T_\gamma \approx 0.2\,\text{MeV}$

Solve analagous EoM for $\delta_\gamma = \dfrac{\rho_\gamma(r,t) - \bar{\rho}_\gamma(t)}{\bar{\rho}_\gamma(t)}$

$$\bar{\rho}_\gamma = \frac{\pi^2}{15}\bar{T}_\gamma^{-4}$$

$\bar{T}_\gamma \equiv 2.72\text{K}$ today

$$\rho_\gamma = \frac{\pi^2}{15}\left(\bar{T}_\gamma + \underbrace{\delta T_\gamma(r,t)}_{\text{temp. variation}}\right)^4$$

define $\dfrac{\delta T_\gamma(\hat{n})}{\bar{T}_\gamma}$ $\quad$ unit vector in sky

$$\hat{n} = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$$

typical size: $\sim 10^{-5}$

Spherical Harmonics

$Y_{\ell m}(\theta, \phi)$ $\quad$ $(\theta,\phi) \to \hat{n}$

$\ell = 0, \infty$ $\quad$ $m \in (-\ell, \ell)$

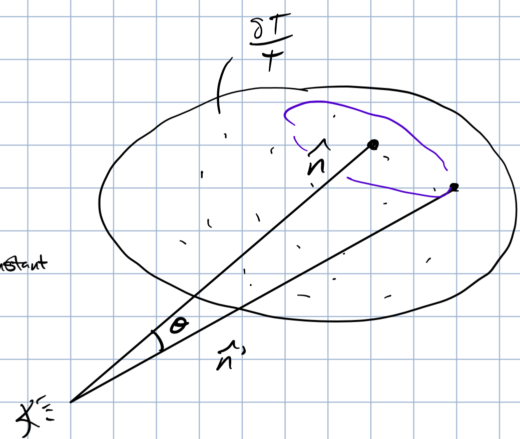
$\nabla^2\psi = 0$ → harmonic

$$Y_{\ell m}(\theta,\phi) = \left(\frac{2\ell+1}{4\pi}\cdot\frac{(\ell-m)!}{(\ell+m)!}\right)^{\frac{1}{2}} \underbrace{P_\ell(\cos\theta)}_{\text{Legendre Polynomial}} \cdot e^{im\phi}$$

$$\oint_{\text{sphere}} d\Omega \, Y_{\ell m} \cdot Y_{\ell' m'} = \delta_{\ell' \ell} \, \delta_{m' m}$$

small ℓ $\rightarrow$ uniform
large ℓ $\rightarrow$ lots of wiggles

$$\frac{\delta T}{T}(\hat{n}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} \cdot Y_{\ell m}(\hat{n})$$



correlation f^{n}: $C(\theta) = \left\langle \frac{\delta T}{T}(\hat{n}) \cdot \frac{\delta T}{T}(\hat{n}') \right\rangle_{\hat{n} \cdot \hat{n}' = \text{constant}}$

avg over all $\hat{n}$'

use $Y_{\ell m}$ identities

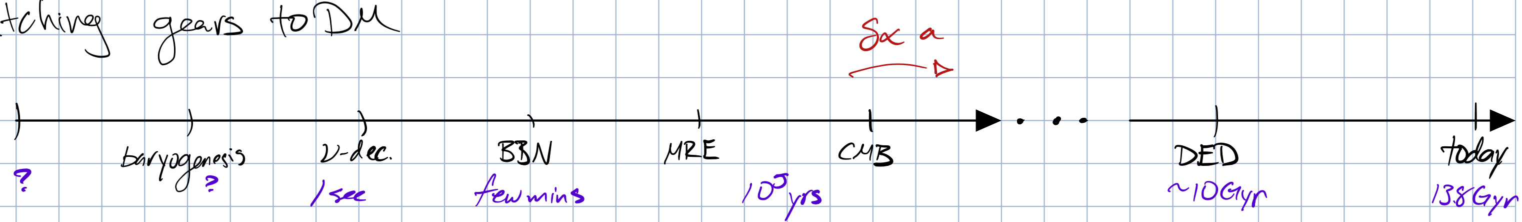
$$C(\theta) = \sum_{\ell=0}^{\infty} \underbrace{C_\ell}_{\propto \sum_{m=-\ell}^{+\ell} |a_{\ell m}|^2} \cdot P_\ell \cdot (\cos\theta)$$

fourier modes

solving EoM gives C_ℓ

= Ratio peaks $\Rightarrow$ Ω_{DM}

Switching gears to DM



Galaxy

M_{gal} – $10^{12} M_\odot$ total mass (stars + gas + DM)
$\sim 10^{58}$ GeV

R_{gal} – 200kpc $\qquad$ pc = $3 \cdot 10^{16}$ m

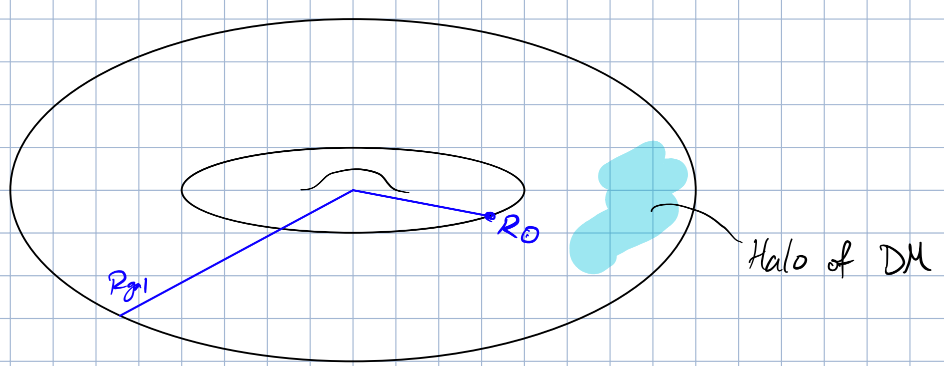
$R_\odot$ – 8 kpc

t_{gal} – 10 Gyr $\qquad \rightarrow$ 50 revs

local density $\quad \delta \sim 10^5$

$\rho^\odot_{DM}(R = R_\odot) = \frac{0.3\ \text{GeV}}{\text{cm}^3}$

$n^\odot_{DM} = \frac{\rho^\odot_{DM}}{m_{DM}}$



DM can't dissipate energy

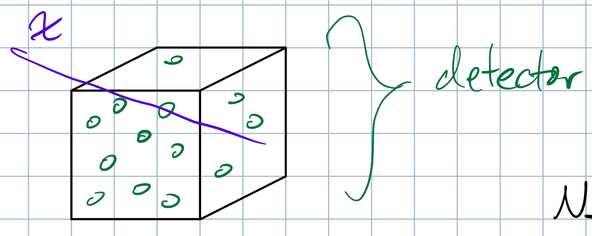
NFW halo:

$$\rho_{DM}(r) = \frac{\rho(r_s)}{\left(\frac{r}{r_s}\right)\left(1+\frac{r}{r_s}\right)^{3-\gamma}}$$

γ – $O(1)$ #

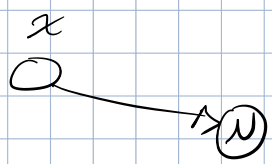
r_s – characteristic radius
diff for each galaxy

Direct Detection

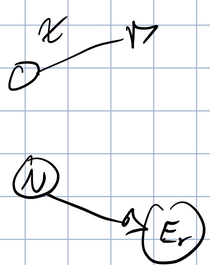


N_T targets nuclei
usually xenon

before:



after:



"recoil energy"
kinetic   observe

ionization
scintillation
heat (SSD)
Cherenkov radiation

need $E_R > E_{thresh} \approx keV$

bombarded by cosmic rays

want rate per nucleus : $R \sim n_\chi \cdot \sigma_\chi \cdot v_{rel}$

$n_\chi = \frac{\rho_\chi^0}{m_\chi}$ cross section   DM velocity

want rate for nucleus of given size

derive $\frac{dR}{dE_R}$

$$\frac{dR}{dE_R} = \underbrace{N_T}_{\#\text{ nuclei}} \cdot \underbrace{n_\chi}_{\text{DM \# density}} \cdot \int \underbrace{d^3v\, f(\vec{v})}_{\text{velocity distribution}} \cdot \frac{d\sigma}{dE_R} \cdot \underbrace{v}_{\text{velocity}}$$

normalize: $\int d^3v\, f(v) = 1$