Grader: Aurora Ireland

how do we go from GeV to SI units?

need context

reduced planck constant

$\hbar = 1.054 \cdot 10^{-34} \frac{kg\, m^2}{s}$

$m_p = 0.938 \frac{GeV}{c^2} = 1.672 \cdot 10^{-27}\, kg \rightarrow kg = 5.60 \cdot 10^{26} \frac{GeV}{c^2}$

$\rightarrow \hbar = 6.583 \cdot 10^{-24}\, GeV\, s$

$GeV^{-1} = 6.583 \cdot 10^{-24}\, s$ time conversion

$c = 2.997 \cdot 10^{8}\, m\, s^{-1} \equiv 1 \rightarrow s = 2.997 \cdot 10^{8}\, m$

$GeV^{-1} = 1.973 \cdot 10^{-14}\, cm$ length conversi

example: CMB

$T_\gamma = 2.34 \cdot 10^{-13}\, GeV$

density $n_\gamma = \frac{2\, \zeta(3)}{\pi^2} T_\gamma^3 = 3.13 \cdot 10^{-39}\, GeV^3$

HW hint

want SI? $\rightarrow n_\gamma = 3.13 \cdot 10^{-39} \cdot (1.973 \cdot 10^{-14})^{-3} \cdot cm^{-3}$

$= 411\ cm^{-3}$

done w/ natural units, now physics!

Cosmic Expansion

Special Relativity Review

conceptual guide

$v \ll 1 \rightarrow$ classical mechanics

$$ds^2 = dx^2 + dy^2 + dz^2 \qquad \text{(non-relativistic)}$$

Galilean Boost

$$\vec{r} \rightarrow \vec{r} + \vec{v} \cdot t$$

$$\vec{r} \rightarrow U \vec{r} \qquad U^T = U^{-1} \qquad U\text{- 3D rotation}$$

$v \sim 1 \rightarrow$ no longer invariant

3D $\rightarrow$ 4D

"four vectors"

Lorentz transformations $\quad 0 \leq \beta \leq 1$

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

define new invariant product: $\quad t'^2 - x'^2 - y'^2 - z'^2 = t^2 - x^2 - y^2 - z^2$

new pythagorean "line element"

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

proper time $\tau \qquad (t, x, y, z) \rightarrow (t+dt,\ x+dx,\ y+dy,\ z+dz)$

set $ds = d\tau$

outside observer

inside observer

$$d\tau^2 = dt^2 \left[1 - \left(\frac{dx}{dt}\right)^2 - \left(\frac{dy}{dt}\right)^2 - \left(\frac{dz}{dt}\right)^2 \right] \qquad dt = \frac{d\tau}{\sqrt{1-v^2}}$$

$$v^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2$$

rewrite!

$$(t\ x\ y\ z) = (x^0, x^1, x^2, x^3)$$

$$ds^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2 \equiv \sum_\mu \sum_\nu \eta_{\mu\nu}\, dx^\mu dx_\nu$$

metric tensor

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$$

General Relativity

principles

① General Coordinate Independence

laws of physics are invariant under arbitrary transformation

② Gravity = Spacetime Curvature

under gravity, free particles travel along shortest distance between points (geodesic)

③ Equivalence Principle

no experiment distinguishes between gravitational & "regular" acceleration

invariant line element

$$ds^2 = \sum_\mu \sum_\nu g_{\mu\nu}(x)\, dx^\mu dx_\nu \equiv g_{\mu\nu}(x)\, dx^\mu dx_\nu$$

$g_{\mu\nu}(x) \equiv$ "position dependent metric tensor"

4×4 object, but $g_{\mu\nu} = g_{\nu\mu} \rightarrow 10$ indpt.

project of GR: calculate metric tensor for given mass/energy solving Einstein field eqⁿ

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

"einstein" tensor

"vacuum energy" can ignore rn

"stress energy tensor"

in vacuum w/ $\Lambda = 0$ & RHS $= 0$ $\rightarrow$ get Special relativity! $g_{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$

Recover Newton

add a weak, non relativistic space

seek sol'ⁿs $g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$ (perturbation)

$G_{00} = 8\pi G T_{00} \rightarrow \nabla^2 h_{00} = 8\pi G\rho$

set $h_{00} \equiv 2\Phi \rightarrow \nabla^2 \Phi = 4\pi G\rho$
(classical mechanics)

Cosmological Expansion

infinitely large isotropic & homogeneous universe (FRW model)
isotropic: look up & its looks same
homogeneous: looks same in all posn

sol'ns $\rightarrow ds^2 = dt^2 - a(t)^2 (dx^2 + dy^2 + dz^2)$

$a(t)$ is dimensionless quantity

$g_{00} = 1, \quad g_{11} = g_{22} = g_{33} = g(t)$

comoving distance: same relative distance
physical distance changes

physical distance: $d(t) = a(t) \cdot r \qquad r = |x_2 - x_1|$

Hubble's law: $\dot{d} = \dot{a} r = \frac{\dot{a}}{a} \cdot (a \cdot r) \equiv Hd, \qquad H \equiv \frac{\dot{a}}{a}$

farther away objects are, faster they move in all directions

observed value $\quad H_0 \simeq 70\ km s^{-1} Mpc^{-1} = 1.4 \cdot 10^{-42}\ GeV$

1 Mpc away, objects move away @ 70 km/s

FRW redshift

$\lambda_0 = \frac{a(t_0)}{a(t_1)} \lambda_1$
observed — emitted

define redshift: $1 + z = \frac{a(t_0)}{a(t_1)}$

momentum: $p = 2\pi f = \frac{2\pi}{\lambda} \propto a^{-1} \qquad (\hbar = 1)$

for any relativistic particle: $E = p \propto a^{-1}$
non-relativistic particle: $E = \sqrt{p^2 + m^2} \simeq m + O(p^2) \propto a^0$

Scale Factor Evolution

mass/energy density must be isotropic & homogeneous $\rightarrow$ $T_{00} = \rho(t)$

into Einstein eq^n:

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3} G\rho \qquad H(t) = \frac{\dot a}{a}$$

given

ignored Λ & assume flat universe

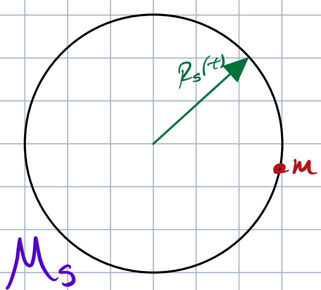
actually measured flat

Λ negligible in early univers

Classical Derivation

Ryden 4.1

constant mass M_s & radius $R_s(t)$ contracting/expanding



"preferred" origin

$$F = -G\frac{M_s m}{R_s(t)^2}$$ force on test mass m

$$\frac{d^2R_s}{dt^2} = -\frac{GM_s}{R_s(t)^2}$$

$$\int\left(\frac{d^2R_s}{dt^2}\right)\left(\frac{dR_s}{dt}\right)\cdot dt = -\int\frac{GM_s}{R_s}\left(\frac{dR_s}{dt}\right)dt$$

$$\frac{1}{2}\left(\frac{dR_s}{dt}\right)^2 = \frac{GM_s}{R_s} + U$$

constant

constant mass: $M_s = \frac{4\pi}{3}\rho(t)\, R_s(t)^3$

isotropic expansion about center: $R_s(t) = a(t)\, r_s$

scale factor

'coord. distance

$$\longrightarrow \frac{1}{2} r_s^2 \cdot \dot a^2 = \frac{4\pi}{3}\cdot G r_s^2\, \rho(t)\cdot a(t)^2 + U$$

set $U=0$ $\rightarrow$ $\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3}G\rho$ exactly @ escape velocity

$E = E$

What is Energy density in representative volume $V(a)$

$$\rho = \frac{1}{V(a)}\cdot\sum_{i=1}^{N}\sqrt{|\vec p_i|^2 + m_i^2}$$

dilutes as a^3

dilutes as a^1

does mass or momentum dominate?

nonrelativistic particle $\to$ $|\vec{p}| \ll m$ $\to$ $\rho \propto a^{-3}$

relativistic particle $\to$ $|\vec{p}| \gg m$ $\to$ $\rho \propto a^{-4}$

volume & energy dilution

$\rho = \sum_j \rho_j$ /particle, / contribution

Matter dominated universe

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\cdot \rho_0 \cdot a^{-3}$$

$a(t_0) \equiv 1$
arbitrary

define $A^2 = \frac{8\pi G}{3}\rho_0$

$$\frac{\dot{a}}{a} = A\cdot a^{-3/2} \to a^{1/2}da = A\cdot dt \to a \propto t^{2/3}$$

if matter is always dominating to present day
$a(t) = \left(\frac{t}{t_0}\right)^{2/3}$

Radiation dominated universe

(up until inflation, 1st hundred thou. yrs)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\cdot \rho_0 \cdot a^{-4}$$

define $A^2 = \frac{8\pi G}{3}\rho_0$

$$\frac{\dot{a}}{a} = Aa^{-2} \to a\,da = A\,dt \to a \propto t^{1/2}$$

if until today, $a(t) = \left(\frac{t}{t_0}\right)^{1/2}$

prof. sounds like Leo Dicaprio in Wolf of Wall Street