last time: H-recombination in equilibrium

$$e^- + p^+ \longrightarrow H + \gamma \qquad B_H = 13.6\,eV$$

match μ's: $\mu_e + \mu_p = \mu_H + \cancel{\mu_\gamma}$ $\qquad T \ll m$

take ratios

$$\left(\frac{n_H}{n_e n_p}\right)_{eq} = \frac{g_H}{\cancel{g_e g_p}} \cdot \left(\frac{m_H}{m_e m_p} \cdot \frac{2\pi}{T}\right)^{3/2} \frac{\exp\left[-\frac{m_H - \cancel{\mu_H}}{T}\right]}{\exp\left[-\frac{m_p - \cancel{\mu_p}}{T}\right]\exp\left[-\frac{m_e - \cancel{\mu_e}}{T}\right]}$$

$m_H \sim m_p$

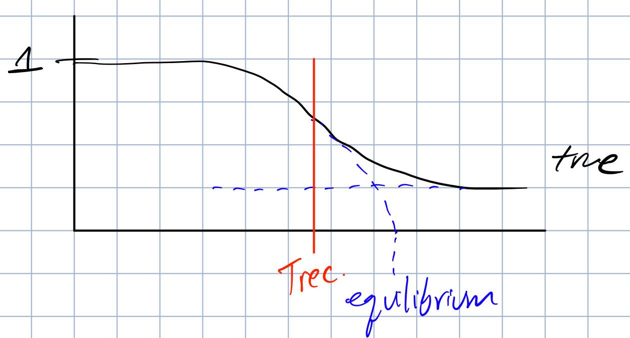
$$= \left(\frac{m_H}{m_e m_p} \frac{2\pi}{T}\right)^{3/2} e^{+B_H/T}$$

$$B_H \equiv m_p + m_e - m_H = 13.6\,eV$$

$$= \left(\frac{2\pi}{m_e T}\right)^{3/2} e^{+B_H/T}$$

ionized fraction $\qquad X_e \equiv \frac{n_e}{n_b} = \frac{n_e}{n_p + n_H}$ $\qquad$ (can ignore neutrons $\rightarrow 1/8$)

X_e starts @ 1 & degrades over time



$T_{recombination} = 0.3\,eV$

charge neutrality: $n_p = n_e$ $\qquad n_H = n_b(1 - X_e)$

$\leftrightarrow 0 \rightarrow$ more

$$n_b = \eta \cdot n_\gamma = \eta \frac{2\zeta(3)}{\pi^2} T^3$$

$$\left(\frac{n_H}{n_e n_p}\right)_{eq} = \left(\frac{n_H}{n_e}\right)^2_{eq}$$

$$\rightarrow \frac{1 - X_e}{X_e^2} = \frac{1 - X_e}{(n_e/n_b)^2} = \frac{n_b^2(1 - X_e)}{n_e^2} = \frac{n_b \cdot n_b(1 - X_e)}{n_e^2}$$

$$= \frac{n_H}{n_e^2} n_b = \frac{n_H}{n_e^2} \cdot \eta \cdot n_\gamma$$

$$\frac{1 - X_e}{X_e^2} = \left(\frac{2\pi}{m_e T}\right)^{3/2} e^{B_H/T} \eta \cdot n_\gamma = \eta \cdot \frac{2\zeta(3)}{\pi^2} \cdot \left(\frac{2\pi T}{m_e}\right)^{3/2} e^{B_H/T}$$

Saha eq^{n}

Timeline ... ν-dec | $e^+e^- \to \gamma\gamma$ | BBN (He formation) | Recombination (H formation) | γ-decoupling (CMB)

$T_\nu \neq T_\gamma$

$T_\gamma \sim MeV$ | $T_\gamma \sim m_e \sim 1/2 MeV$ | $T \sim keV$ | $T \sim 0.3 eV$ | $T \sim eV$

(matter domination)

CMB formation

$e^- + \gamma \leftrightarrow e^- + \gamma$

photons can't go far... till now!

$$\rho_{TOT} = \underbrace{\rho_\nu + \rho_\gamma}_{\text{relativistic}} + \underbrace{\rho_{DM} + \rho_B + \rho_e}_{\text{nonrelativistic}} \approx 6\rho_B \qquad T \ll m$$

$\rho_\nu + \rho_\gamma \ll \rho_{DM} + \rho_B + \rho_e$

$\rho_B \gg \rho_e$

Hubble rate: $H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3} G \rho_{TOT}$

$$= \frac{8\pi}{3} G (m_p \cdot n_b \cdot 6)$$

$$= 16\pi G\, m_p \cdot \eta \cdot n_\gamma$$

$$= 16\pi G\, m_p \cdot \eta \cdot \left(\frac{2\zeta(3)}{\pi^2} T^3\right) = G \frac{32}{\pi} \cdot \eta \cdot \zeta(3) \cdot m_p T^3 = H^2$$

Rate of reaction: when can it go 1 Hubble time before hitting another e^-?

$R_\gamma = n_E \cdot \sigma_T$ $\qquad \sigma_T = \frac{8\pi}{3} \frac{\alpha^2}{m_e^2} \qquad \alpha = 1/137$

$\quad$ Thomson scattering

$= \eta \cdot \left(\frac{2\zeta(3)}{\pi^2} T^3\right) \cdot \left(\frac{8\pi}{3} \frac{\alpha^2}{m_e^2}\right)$

(if all e^-'s are free)

not true tho b/c H's are being formed

ion-charged particle

$R_\gamma \Longrightarrow R_\gamma^{ion} = X_E \cdot R_\gamma$

set $H \approx R_\gamma^{ion}$

$$\left(\frac{32}{\pi} G \eta \zeta(3) m_p T^3\right)^{1/2} \doteq X_e \left(\eta \frac{2\zeta(3)}{\pi^2} \cdot T^3\right) \cdot \left(\frac{8\pi}{3} \frac{\alpha^2}{m_e^2}\right)$$

$T_{\gamma\text{-dec}} \approx 0.27\,eV$

about 390,000 years after Big Bang

① ok - but dark matter
② non equilibrium physics ☆

Boltzman Equation

$$f_{eq}(p,t) = \frac{g}{\exp[-\frac{E-\mu}{T}] \pm 1}$$ max entropy

$$N = \int dx^3 \frac{d^3p}{(2\pi)^3} f(\vec{p}, \vec{x}, t)$$ f - phase space distribution

$$n = \int \frac{d^3p}{(2\pi)^3} f(p, x, t)$$ isotropic ∃ homogeneous Universe

left hand side $\frac{d}{dt}[f(p,t)] = \frac{\partial f}{\partial t} + \frac{\partial p}{\partial t} \cdot \frac{\partial f}{\partial p}$ $\quad p(t) = \frac{p_0}{a(t)}$ why inverse 1? why not $\frac{p}{a^2}$

$$= \frac{\partial f}{\partial t} - H \cdot p(t) \cdot \frac{\partial f}{\partial p} \qquad \frac{\partial p}{\partial t} = -\frac{\dot a}{a^2} p_0$$

integrate over $d^3p/(2\pi)^3$

$$\frac{\partial p}{\partial t} = -\left(\frac{\dot a}{a}\right) \cdot \frac{p_0}{a} = -H \cdot p(t)$$

$$\int \frac{d^3p}{(2\pi)^3} \cdot \left(\frac{df}{dt}\right) = \int \frac{d^3p}{(2\pi)^3} \cdot \left(\underline{\frac{\partial f}{\partial t}} - \underline{Hp \frac{\partial f}{\partial p}}\right)$$

$$\frac{\partial}{\partial t} \int \frac{d^3p}{(2\pi)^3} f = \frac{\partial}{\partial t}(n(t)) = \frac{dn}{dt}$$

integrate by parts

$$-H \frac{1}{2\pi^2} \int_0^\infty dp\, p^3 \frac{\partial f}{\partial p} = 3H \frac{1}{2\pi^2} \left(\int dp\, p^2 f(p,t)\right) = 3Hn$$

$$\frac{dn}{dt} + 3Hn = \frac{d}{dt}[f(p,t)]$$

if RHS = 0 $\longrightarrow \frac{dn}{dt} = -3Hn \longrightarrow \frac{dn}{n} = -3H\,dt = -3\,\frac{1}{a}\cancel{\frac{da}{dt}dt}$

$$\frac{dn}{n} = -3\frac{da}{a} \longrightarrow \ln(n) = \ln(a^{-3}) + C = \log(c_2 \cdot a^{-3})$$

$$n(a) = n(a_i) \cdot \left(\frac{a_i}{a}\right)^3$$

if $RHS \neq 0$ $\quad 1+2 \longleftrightarrow 3+4$

$$\frac{dn_1}{dt} + \underbrace{3Hn_1}_{\text{Hubble expansion}} = \underbrace{-An_1n_2}_{\text{destruction rate}} + \underbrace{Bn_3n_4}_{\text{creation rate}}$$

in equilibrium: $A\, n_1^{eq} n_2^{eq} = B\, n_3^{eq} n_4^{eq}$

$$B = A \cdot \left(\frac{n_1^{eq} n_2^{eq}}{n_3^{eq} n_4^{eq}} \right)$$

$A \equiv \langle \sigma \cdot v \rangle$ thermally avg x-section x velocity

$$\frac{dn_1}{dt} + 3Hn_1 = -\langle \sigma \cdot v \rangle \left(n_1 n_2 - \left(\frac{n_1^{eq} n_2^{eq}}{n_3^{eq} n_4^{eq}} \right) n_3 n_4 \right)$$

particle physics/QFT

$$n_i^{eq} = \int \frac{d^3p}{(2\pi)^3} \frac{g}{e^{\frac{-(E-\mu)}{T}} \pm 1}$$

Apply to $e^- + p^+ \longleftrightarrow H + \gamma$

$n_e = n_p$
$n_H \approx n_{H,eq}$
$n_{\gamma,eq} = \frac{2\zeta(3)}{\pi^2} T^3$

$X_e \equiv \frac{n_e}{n_p + n_H} = \frac{n_e}{n_b}$ $\qquad n_b = \eta\, n_\gamma^{eq}$

free electron

$$\frac{1}{a^3} \cdot \frac{d(a^3 n_e)}{dt} = -\langle \sigma \cdot v \rangle \cdot \left(n_e n_p - \left(\frac{n_e^{eq} n_p^{eq}}{\cancel{n_H^{eq}}\ \cancel{n_\gamma^{eq}}} \right) \cancel{n_H}\ \cancel{n_\gamma} \right)$$

$n_e = n_p$

$n_e = n_p \approx n_b \cdot X_e$

$$= -\langle \sigma \cdot v \rangle \left(n_e^2 - n_{e,eq}^2 \right)$$

b/c $n_b = \eta \cdot n_\gamma \propto a^{-3}$

$n_e = n_b \cdot X_e$

$\longrightarrow a^3 \cdot n_b = \text{const.}$

can bring outside

$$\frac{1}{a^3} \frac{d(X_e\, n_b a^3)}{dt} = \frac{n_b \cdot a^3}{a^3} \frac{dX_e}{dt} = n_b \cdot \frac{dX_e}{dt}$$

Saha eq$^{\underline{n}}$

$$n_b \cdot \frac{dX_e}{dt} = -\langle \sigma \cdot v \rangle \left(n_b^2 X_e^2 - (n_{b,eq})^2 (X_{e,eq})^2 \right)$$

$$\frac{dX_e}{dt} = -\langle \sigma \cdot v \rangle\, n_b \left(X_e^2 - X_{e,eq}^2 \right)$$

$\hookrightarrow \eta \cdot n_\gamma \propto T^3$ $\qquad T \propto \frac{1}{a(t)}$

$$\langle \sigma \cdot v \rangle \approx \sigma_T \cdot \left(\frac{B_H}{T} \right)^{3/2}$$

$\frac{8\pi}{3} \frac{\alpha^2}{m_e^2}$