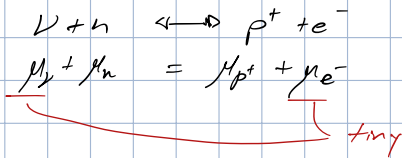


initial n/p ratio set by T_{dec} .



b/c $T \gg m_e, m_\nu$

$$n_{e^-} - n_{e^+} = \frac{g_{e^-}}{2\pi} T^3 \left(\pi^2 \frac{\mu_e}{T} + \left(\frac{\mu_e}{T}\right)^3 \right)$$

by charge conservation $n_{e^-} + n_{e^+} = n_p - n_{\bar{p}} \approx \frac{1}{2} n_{\nu}$

can ignore e^- & ν

initial conditions for BBN: $\nu + n \leftrightarrow p^+ + e^-$, stops @ $T_{dec} \approx 0.8 \text{ MeV}$

$$\left(\frac{n_n}{n_p}\right)_{eq} = \left(\frac{g_n}{g_p}\right) \left(\frac{\mu_n}{\mu_p}\right)^{3/2} \left(\exp[-(m_n - \mu_n)/T] / \exp[-(m_p - \mu_p)/T] \right)$$

$\mu_p = \mu_n$

$$= e^{-Q/T}$$

$Q \equiv m_n - m_p \approx 1.3 \text{ MeV}$

$$X_n(t) = \frac{n_n}{n_p + n_n} = X_n(t_{dec}) \cdot e^{-t/\tau}$$

$\tau \approx 10$ $\tau = 900s$ $t = 2400s$

Expansion rate, radiation dominated

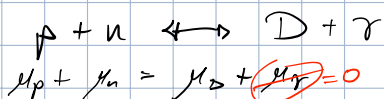
$$H = 1.66 \sqrt{g_*} \frac{T^2}{m_{pl}}$$

Reaction rates (lab)

$$\eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = ?? \quad 6 \cdot 10^{-10}$$

favors high binding energy \rightarrow why not iron? at some time, reactions stop equilibrium is to get #s within order of magnitude

Deuterium Formation



$$\left(\frac{n_D}{n_p \cdot n_n}\right)_{eq} = \frac{g_D}{g_p \cdot g_n} \left(\frac{\mu_D}{\mu_p \mu_n} \frac{2\pi}{T}\right)^{3/2} \frac{\exp[-(m_D - \mu_D)/T]}{\exp[-(m_p - \mu_p)/T] \exp[-(m_n - \mu_n)/T]}$$

$m_D = 2m_p$

$$= \frac{3}{4} \left(\frac{4\pi}{m_p \cdot T} \right)^{3/2} e^{B_D/T}$$

grows w/ time

binding energy

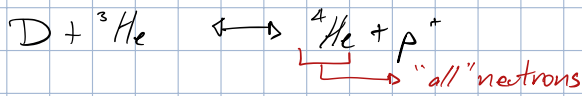
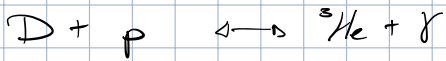
$$B_D \equiv m_n + m_p - m_D \approx 2.2 \text{ MeV}$$

$$\left(\frac{n_D}{n_p} \right)_{eq} \propto \left(\frac{n_n}{n_p} \right)_{eq} e^{B_D/T}$$

↘ $\propto \eta$

Wait until $t_{nuc} = \frac{1}{2H(T_{nuc})} \approx 330 \text{ sec}$

Deuterium forms

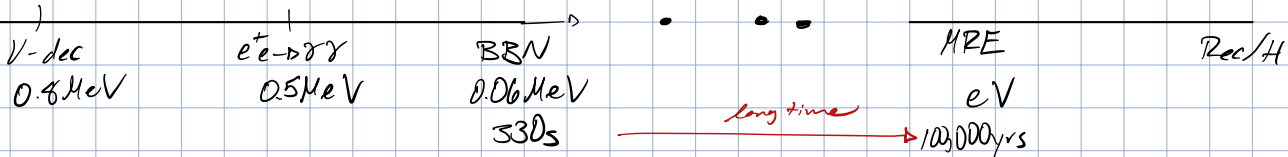


How many neutrons?

$$X_n(t_{nuc}) = \frac{1}{6} e^{-t_{nuc}/\tau_n} \approx \frac{1}{8}$$

of neutrons @ t_{nuc}

$$Y_p \equiv \frac{p_{He}}{p_p} = 4X_n(t_{nuc}) \approx \frac{1}{4}$$



Matter Radiation Equality

until now: radiation dominated universe

$$H^2 = \frac{8\pi}{3} G_{rad}$$

$$\rho_{rad} = \rho_\gamma + \rho_\nu \quad @ \quad T \lesssim \text{MeV}$$

$$H^2 = \frac{8\pi}{3} G \left(\frac{\pi^2}{30} g_*(T) T^4 \right)$$

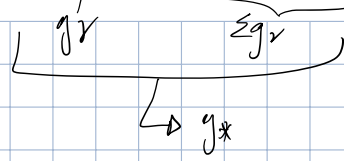
$$\rho_\gamma = \frac{\pi^2}{30} g_\gamma T_\gamma^4$$

$$H = 1.66 \sqrt{g_*} \frac{T^2}{m_{pl}}$$

$$\rho_\nu = \frac{\pi^2}{30} \cdot \left(\frac{7}{8} \right) \cdot (3 \cdot 2) T_\nu^4$$

$$T_\nu = \left(\frac{4}{11} \right)^{1/3} T_\gamma$$

$$\rho_{rad} = \frac{\pi^2}{30} \left(2 + \frac{7}{8} (3 \cdot 2) \cdot 6 \left(\frac{T_\nu}{T_\gamma} \right)^4 \right) T_\gamma^4$$



also matter!

$$\rho_M = \rho_B + \rho_{DM}$$

$$\rho_B = \rho_p + \rho_n \quad \text{protons \& neutrons}$$

$$\rho_{DM} \approx 5\rho_B$$

$$\rho_M = 6\rho_B \approx 6 \cdot \rho_p \quad (\rho_n \text{ decays fast})$$

$$\rho_M = 6 \cdot m_p \cdot n_p = 6 m_p (\eta \cdot n_\gamma) = 6 m_p \cdot \eta \cdot \left(\frac{2 \cdot j(s)}{\pi^2} T_\gamma^3\right) = 12 \frac{m_p \eta}{\pi^2} \sqrt[1.2]{j(s)} T_\gamma^3$$

$$\rho_R = \frac{\pi^2 g_*}{80} T_\gamma^4 \quad \text{for } T < m_e$$

equality $\rho_M = \rho_R$

$$\frac{\pi^2 g_*}{80} T_\gamma^4 = \frac{12 m_p \eta}{\pi^2} (1.2) T_\gamma^3$$

$$\rightarrow T_\gamma = \frac{150 \cdot m_p \cdot \eta}{\pi^4 \cdot g_*} (1.2) \approx eV$$

3.58

matter domination changes how matter accretes in gravity potential

$\gamma \rightarrow$ gravity dimple \rightarrow redshifted \rightarrow temp. variation \rightarrow CMB

*to agree w/ CMB
need this created
@ specific time & $\rho_{DM} \approx 5\rho_B$*

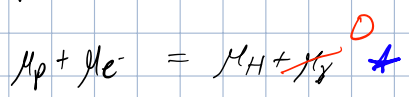
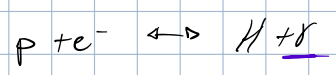
how much single electrons left after H made?

Recombination - H formation

$$T_{rec} \approx 0.1 eV$$

$$H: B_{H1} = 13.6 eV$$

can't happen before, soooo many γ to smash you & break you up (set by η)



M_e no longer relativistic
 $T \ll m_e \quad T \approx 0.1 eV$

for counting: $n_e = n_p \approx \eta \cdot n_\gamma$

neutral charge: $n_{e^-} - n_{e^+} = n_p - n_{p^+}$

H-atom abundance

$\rightarrow n_p = n_{e^-}$
*not enough \bar{p} to create things, all annihilated
 \therefore 1/2 matter/antimatter difference*

$$\left(\frac{n_H}{n_p \cdot n_e}\right)_{eq} = \frac{g_H}{g_p \cdot g_e} \left(\frac{m_H}{m_p m_e} \frac{2\pi}{T}\right)^{3/2} \frac{\exp\left[-\frac{(m_H - M_H)}{T}\right]}{\exp\left[-\frac{(m_p - M_p)}{T}\right] \exp\left[-\frac{(m_e - M_e)}{T}\right]}$$

$m_H \approx m_p$ in coefficient

$$\frac{n_H}{n_p \cdot n_e} = \frac{n_H}{(n_e)^2}$$

$$\left(\frac{n_H}{(n_e)^2}\right)_{eq} = \left(\frac{2\pi}{m_e}\right)^{3/2} \cdot \exp\left[-\frac{m_p + m_e - m_H}{T}\right]$$

$$B_H = m_p + m_e - m_H = 13.6 \text{ eV}$$

$$\left(\frac{n_H}{(n_e)^2}\right)_{eq} = \frac{2\pi}{m_e} e^{B_H/T}$$

$n_e \approx 10^{-9}$

need to wait long time
to have B_H compete w/ n_e

$$X_e(t) = \frac{n_e}{n_p} = \frac{n_e}{n_p + n_n + n_H} \approx \frac{n_e}{n_p + n_H}$$

$$n_e = \eta \cdot n_T = \eta \cdot \frac{2\zeta(3)}{\pi^2} \cdot T^3$$

$$\left(\frac{1-X_e}{X_e^2}\right) = \eta \cdot \left(\frac{2\zeta(3)}{\pi^2}\right) \left(\frac{2\pi}{m_e} T^3\right) e^{-\frac{B_H}{T}}$$

Saha equation