initial n/p ratio set by T_{dec}.

$$\nu + n \longleftrightarrow p^+ + e^-$$

$$\underline{\mu_\nu} + \mu_n = \mu_{p^+} + \underline{\mu_{e^-}}$$

tiny

b/c $T \gg m_e,\ m_\nu$

$$n_{e^-} - n_{e^+} = \frac{g_*}{2\pi} T^3 \left(\pi^2 \frac{\mu_e}{T} + \left(\frac{\mu_e}{T}\right)^3 \right)$$

by charge conservation $n_{e^-} + n_{e^+} = n_p - n_{\bar{p}} \simeq \frac{1}{2} \eta\, n_\gamma$

can ignore e^- & ν

initial conditions for BBN: $\nu + n \longleftrightarrow p^+ + e^-$, stops @ $T_{dec} \simeq 0.8\,\text{MeV}$

$$\left(\frac{n_n}{n_p}\right)_{eq} = \underbrace{\left(\frac{g_n}{g_p}\right)}_{1} \underbrace{\left(\frac{m_n}{m_p}\right)}_{\sim 1}^{3/2} \left(\exp[-(m_n - \cancel{\mu_n})/T] \big/ \exp[-(m_p - \cancel{\mu_p})/T] \right)$$

$\mu_p = \mu_n$

$$= e^{-Q/T} \qquad Q \equiv m_n - m_p \approx 1.3\,\text{MeV}$$

$$X_n(t) = \frac{n_n}{n_p + n_n} = \underbrace{X_n(t_{dec})}_{\sim 1/6} \cdot e^{-t/\tau} \qquad t = \tfrac{1}{2H(t)}$$

$\tau = 900\text{s}$

Expansion rate, radiation dominated

$$H = 1.66 \sqrt{g_*}\, \frac{T^2}{m_{pl}}$$

Reaction rates (lab)

$$\eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = ?? \qquad 6 \cdot 10^{-10}$$

favors high binding energy $\rightarrow$ why not iron? at some time, reactions stop

equillibrium is to get #'s within order of magnitude

Deuterium Formation

$$p + n \longleftrightarrow D + \gamma$$

$$\mu_p + \mu_n = \mu_D + \mu_\gamma = 0$$

$$\left(\frac{n_D}{n_p \cdot n_n}\right)_{eq} = \frac{g_D}{g_p \cdot g_n} \cdot \left(\frac{\overset{\approx 2m_n}{m_D}}{m_p m_n} \frac{2\pi}{T} \right)^{3/2} \frac{\exp\left[-\frac{(m_D - \mu_D)}{T}\right]}{\exp\left[-\frac{m_p - \cancel{\mu_p}}{T}\right] \exp\left[-\frac{(m_n - \cancel{\mu_n})}{T}\right]}$$

$m_D = 2m_p$

$$= \frac{3}{4}\left(\frac{4\pi}{m_p \cdot T}\right)^{3/2} \underbrace{e^{B_D/T}}_{\text{grows w/ time}} \qquad \overbrace{B_D}^{\text{binding energy}} \equiv m_n + m_p - m_D \approx 2.2\,\text{MeV}$$

$$\left(\frac{n_D}{n_p}\right)_{eq} \propto \underbrace{(n_n)_{eq}}_{\propto\, \eta}\, e^{B_D/T}$$

Wait until $t_{nuc} = \frac{1}{2H(T_{nuc})} \approx 330\,\text{sec.}$

Deuterium forms

$$D + p \longleftrightarrow {}^3He + \gamma$$

$$D + {}^3He \longleftrightarrow \underbrace{{}^4He}_{\text{"all" neutrons}} + p^+$$

How many neutrons?

$$X_n(t_{nuc}) = \tfrac{1}{6}\, e^{-t_{nuc}/890s} \approx 1/8$$

$\#$ of neutrons @ t_{nuc}

$$Y_p = \frac{\rho_{He}}{\rho_P} = 4X_n(t_{nuc}) \approx 1/4$$

ν-dec	$e^+e^- \to \gamma\gamma$	BBN		MRE	Rec/H
0.8 MeV	0.5 MeV	0.06 MeV		eV	
		330 s	long time $\longrightarrow$	100,000 yrs	

Matter Radiation Equality

until now: radiation dominated universe

$$H^2 = \frac{8\pi}{3} G \rho_{rad} \qquad \rho_{rad} = \rho_\gamma + \rho_\nu \quad @\ T \lesssim \text{MeV}$$

$$H^2 = \frac{8\pi}{3} G \left(\frac{\pi^2}{30} g_*(T) \cdot T^4\right) \qquad \rho_\gamma = \frac{\pi^2}{30} g_* T_\gamma^4$$

$$H = 1.66\sqrt{g_*}\,\frac{T^2}{m_{pl}} \qquad \rho_\nu = \frac{\pi^2}{30}\cdot\left(\frac{7}{8}\right)\cdot(3\cdot 2)\, T_\nu^4 \qquad T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

$$\rho_{rad} = \frac{\pi^2}{30}\left(2 + \frac{7}{8}(3\cdot 2)\cdot 6\left(\frac{T_\nu}{T_\gamma}\right)^4\right) T_\gamma^4$$

$$\underbrace{g_\gamma \qquad \sum g_\nu}_{g_*}$$

also matter!

$$\rho_M = \rho_B + \rho_{DM}$$

$\rho_B = \rho_p + \rho_n$ protons + neutrons

$\rho_{DM} \approx 5\rho_B$

$$\rho_M = 6\rho_B \approx 6 \cdot \rho_p \qquad (\rho_n \text{ decays fast})$$

$$\rho_M = 6 \cdot m_p \cdot n_p = 6 m_p (\eta \cdot n_\gamma) = 6 m_p \cdot \eta \cdot \left(\frac{2\,\zeta(3)}{\pi^2} T_\gamma^3\right) = 12 \frac{m_p \eta}{\pi^2} \overbrace{\zeta(3)}^{1.2} T_\gamma^3$$

$$\rho_R = \frac{\pi^2 g_*}{30} T_\gamma^4 \qquad \text{for} \quad T < m_e$$

equality $\rho_M = \rho_R$

$$\frac{\pi^2 g_*}{30} T_\gamma^4 = \frac{12\, m_p \cdot \eta}{\pi^2} (1.2) T_\gamma^3$$

$$\longrightarrow T_\gamma = \frac{150 \cdot m_p \cdot \eta}{\pi^4 \cdot g_*} (1.2) \simeq \text{eV}$$

3.38

matter domination changes how matter accretes in gravity potential

$$\gamma \longrightarrow \underbrace{\text{gravity dimple}} \longrightarrow \text{redshifted} \longrightarrow \text{temp. variation} \longrightarrow \text{CMB}$$

to agree w/ CMB
need this created
@ specific time & $\rho_{DM} \simeq 5\rho_B$

how much single electrons left after H made?

Recombination - H formation

$T_{rec} \approx 0.1\,\text{eV}$

H: $B_H = 13.6\,\text{eV}$

can't happen before, soooo many γ to smash you & break you up (set by η)

$$p + e^- \longleftrightarrow H + \gamma$$

$$\mu_p + \mu_{e^-} = \mu_H + \cancel{\mu_\gamma}^{\,0} \quad *$$

μ_{e^-} no longer relativistic

$T \ll m_e$ $\quad T \approx 0.1\,\text{eV}$

for counting: $n_e = n_p \approx \eta \cdot n_\gamma$

neutral charge: $n_{e^-} - \cancel{n_{e^+}} = n_p - \cancel{n_{\bar{p}}}$

not enough $\bar{p}$ to create things, all annihilated b/c matter/antimatter difference

$$\longrightarrow n_p = n_{e^-}$$

H-atom abundance

$$\left(\frac{n_H}{n_p \cdot n_e}\right)_{eq} = \frac{\overset{4}{g_H}}{\underset{2}{g_p} \cdot \underset{2}{g_e}}\;1 \left(\frac{m_H}{m_p m_e}\,\frac{2\pi}{T}\right)^{3/2} \cdot \frac{\exp\left[\frac{-(m_H - \mu_H)}{T}\right]}{\exp\left[\frac{-(m_p - \mu_p)}{T}\right] \exp\left[\frac{-(m_e - \mu_e)}{T}\right]} \; *$$

$m_H \approx m_p$ in coffient

$$\frac{n_H}{n_p \cdot n_e} = \frac{n_H}{(n_e)^2}$$

$$\left(\frac{n_H}{(n_e)^2}\right)_{eq} = \left(\frac{2\pi}{m_e}\right)^{3/2} \cdot \exp\left[\frac{m_p + m_e - m_H}{T}\right]$$

$B_H = m_p + m_e - m_H = 13.6\,eV$

$$\left(\frac{n_H}{(n_e)^2}\right)_{eq} = \frac{2\pi}{m_e}\; e^{B_H/T}$$

$n_e \propto \eta \sim 10^{-9}$

need to wait long time to have B_H compete w/ n_e

$$X_e(t) = \frac{n_e}{n_B} = \frac{n_e}{n_p + n_n + n_H} \approx \frac{n_e}{n_p + n_H}$$

$$n_e = \eta \cdot n_\gamma = \eta \cdot \frac{2\zeta(3)}{\pi^2} \cdot T_\gamma^3$$

$$\left(\frac{1 - X_e}{X_e^2}\right) = \eta \cdot \left(\frac{2\zeta(3)}{\pi^2}\right)\left(\frac{2\pi}{m_e} T_\gamma^3\right) e^{\frac{B_H}{T_\gamma}}$$

Saha equation