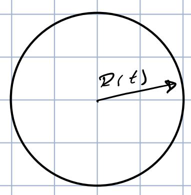
last time



$$\rho(t) = \underbrace{\bar{\rho}(t)}_{\propto a^{-3}} \; (1 + \underbrace{\delta(t)}_{|\delta| \ll 1})$$

Newton's 2^{nd} Law: $\quad \ddot{R} = -\frac{G}{R^2} M = -\frac{G}{R^2}\left(\frac{4\pi}{3} R(t)^3 \rho(t)\right)$

$$= -\frac{4\pi}{3} G \cdot R(t) \, \bar{\rho}(t) \, (1+\delta(t))$$

Mass conservation $\quad M = \frac{4\pi}{3} R(t)^3 \bar{\rho}(t) \, (1+\delta(t)) = \text{const}$

write $\frac{\ddot{R}}{R}$ 2 different ways $\rightarrow$ "equation of motion" for density

$\rightarrow \delta(\vec{r}, t)$

General EOM for $\delta(t)$

$$\star \quad \ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}\Omega_M H^2 \delta = 0 \quad \star$$

(1) Radiation domination $\quad (T \gg eV, \; t \gg 10^6 \, yrs)$

for simplicity, $T \gg m_t \approx 175 \, GeV$, $\quad g_*(T) = \text{const} = 106.75$

$$\Omega_M = \frac{\rho_{DM} + \rho_B}{\rho_{tot}} \approx \rho_{DM}$$

$H^2 = \frac{8\pi}{3} G \rho_R = \frac{8\pi}{3} G \left(\frac{\pi^2 g_*}{30} T^4\right) \rightarrow H \propto a^{-2}$

$H = \frac{1}{a} \cdot \frac{da}{dt} = B a^{-2} \rightarrow a\,da = B\,dt \rightarrow \int a\,da = B\int dt$

$\frac{a^2}{2} = B \cdot t \qquad a(t) \propto t^{1/2}$

$H = \frac{\dot{a}}{a} = \frac{(1/2)t^{-1/2}}{t^{1/2}} = \frac{1}{2t}$

b/c in RD $\Omega_M \approx 0$

$\rightarrow \ddot{\delta} + 2\left(\frac{1}{2t}\right)\dot{\delta} = 0$

$\frac{d^2\delta}{dt^2} = \frac{-1}{t} \cdot \frac{d\delta}{dt} \rightarrow \frac{d^2\delta}{(d\delta/dt)} = -\frac{dt}{t}^2$

$$\left(\frac{d^2\delta}{d\delta}\right) dt = -\frac{dt^3}{t}$$

$$d\delta = -\frac{dt}{t}$$

$$\delta(t) = B_1 + B_2 \log(t)$$ General solⁿ in RD

② Matter Domination

$$H^2 = \frac{4\pi}{3} G\rho \quad \propto a^{-3}$$

$$H = A \cdot a^{-3/2} = \frac{da}{dt}\frac{1}{a}$$

$$A\,dt = a^{1/2}\,da \quad \int$$

$$At = \frac{2}{3} a^{3/2}$$

$$a(t) \propto t^{2/3} \quad \text{in MD}$$

$$H = \frac{\frac{2}{3}t^{-1/3}}{t^{2/3}} = \frac{2}{3t}$$

b/c in MD, $\Omega_M \approx 1$

$$\ddot{\delta} + 2\left(\frac{2}{3t}\right)\dot{\delta} - \frac{3}{2}\left(\frac{2}{3t}\right)^2 \delta = 0$$

hard. guess polynomial $\delta(t) = A \cdot t^n$ $\quad \dot{\delta} = Ant^{n-1}$ $\quad \ddot{\delta} = An(n-1)t^{n-2}$

$$\frac{d^2\delta}{dt^2} + \frac{4}{3t}\frac{d\delta}{dt} - \frac{2}{3t^2}\delta = 0$$

$$An(n-1)t^{n-2} + \frac{4}{3t} \cdot Ant^{n-1} - \frac{2}{3t^2} At^n = 0$$

factor out t^{n-2}

$$At^{n-2}\left(n(n-1) + \frac{4}{3}n - \frac{2}{3}\right) = 0$$

$$n^2 - n + \frac{4}{3}n - \frac{2}{3} = 0$$

$$n^2 + \frac{1}{3}n - \frac{2}{3} = 0$$

$$n = \frac{-\frac{1}{3} \pm \sqrt{(\frac{1}{3})^2 - 4(1)(-\frac{2}{3})}}{2 \cdot (1)}$$

$$= \frac{-\frac{1}{3} \pm \sqrt{\frac{1}{9} + \frac{8}{3}}}{2} = \frac{-\frac{1}{3} \pm \sqrt{\frac{1}{9} + \frac{24}{9}}}{2}$$

$$= \frac{-1/3 \pm \sqrt{25/9}}{2} = \frac{-1/3 \pm 5/3}{2} = \frac{1}{6}(-1 \pm 5)$$

$n = \frac{1}{6}(-1 \pm 5)$

general sol$^{\underline{n}}$ should be linear comb. of both

$n_1 = \frac{1}{6}(-1+5) = 2/3$ $\qquad n_2 = \frac{1}{6}(-1-5) = -1$

$\delta(t) = \underbrace{A_1 t^{2/3}}_{\text{grow}} + \underbrace{A_2 t^{-1}}_{\text{decay} \approx 0}$

in MD, $a(t) \propto t^{2/3}$

$\rightarrow \delta(t) \propto a(t)$

③ Dark Energy Domination $\qquad t \sim 10$ Gyrs

DE doesn't redshift

$\rho_\Lambda \propto a^0$ $\qquad$ no energy conservation $\rightarrow$ mass conservation tho? is local

energy not conserved is global

$H^2 = \frac{8\pi}{3} G \rho_\Lambda = \text{const} \equiv \left(\frac{\Lambda^2}{2}\right)^2$

$\frac{da}{dt}\frac{1}{a} = Q \rightarrow \frac{da}{dt} = Qa \rightarrow a(t) = e^{Qt}$

$\frac{\dot{a}}{a} = Q$

$\rightarrow \ddot{\delta} + 2Q\dot{\delta} = 0$ $\quad \int$

$\dot{\delta} + 2Q\delta = 0$

$\frac{d\delta}{dt} = -2Q\delta \rightarrow \delta(t) = Ce^{-Qt}$

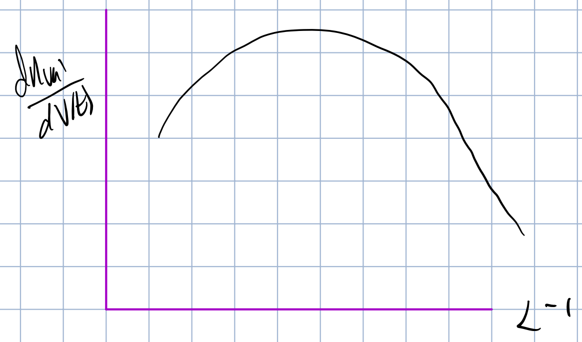
perturbations decay $\quad \delta(t) = Ce^{-H \cdot t}$

Radiation Domination: $\delta(t) = B_2 \ln(t) + B_1$

Matter Domination: $\delta(t) = A_1 t^{2/3} \propto a$

Dark Energy Domination: $\delta(t) = C e^{-H \cdot t}$

Matter Power Spectrum

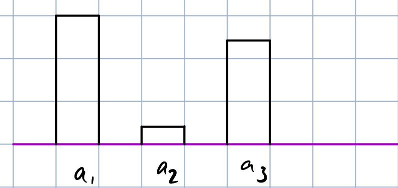


Fourier Transform: generalization of Fourier series

series: $f(x) = \sum a_n \cos\left(\frac{n\pi}{L} x\right)$ defined in interval L

goal is to determine a_n

other form: $f(x) = \sum b_n e^{i\frac{n\pi}{L}x}$



a's are binned

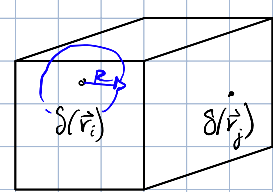
transform: continuous w/o defined limit

x – pos^{n} space
a_n – fourier space

continuous in fourier transform

2 Point Correlation f^{n} $C(\vec{R})$

we have $\delta(\vec{r}, t)$



$$C(\vec{R}) = \Big\langle \sum_i \underbrace{\delta(\vec{r}_i)}_{\frac{\rho(\vec{r}_i) - \bar{\rho}}{\bar{\rho}}} \, \delta(\vec{r}_i + \vec{R}) \Big\rangle$$

angular avg

$$C(\vec{\rho}) = C(|\vec{R}|)$$