$\cdots$ ——— ν-dec ——— e^+e^- annihilation ——— BBN ———▶ $\cdots$ MRE $\cdots$ γ-dec., H-formation, CMB

ν-dec: $T_{dec} \approx 0.8\,\text{MeV}$

e^+e^- annihilation: $T \approx m_e \sim 0.5\,\text{MeV}$

BBN: $T = 100\,\text{keV} = 0.1\,\text{MeV}$

MRE: 100,000 years

γ-dec., H-formation, CMB: 300,000 years

Last time: ν-decoupling

$\nu e^- \longleftrightarrow \nu e^-$ (weak force), $\gamma e \longleftrightarrow \gamma e$ (E&M)

$\nu p^+ \longleftrightarrow n e^+$

$\nu n \longleftrightarrow p^+ e^-$

$n_{p,n} \propto \eta \approx 10^{-9}$

$T \approx 0.8\,\text{MeV}$

$e^+e^- \longrightarrow \gamma\gamma \qquad T \approx m_e$

$$S = \frac{2\pi^2}{45} g_{*,s} T_\gamma^3$$

$$S_i = \frac{2\pi^2}{45}\left(2 + \frac{7}{8}(2\cdot 2 + 3\cdot 2\cdot 1)\right) T_{\gamma_i}^3$$

$$S_f = \frac{2\pi^2}{45}\left(2 + \frac{7}{8}(3\cdot 2\cdot 1)\left(\frac{T_\nu}{T_{\gamma f}}\right)^3\right) T_{\gamma f}^3$$

$$T_{\gamma i} = T_{\nu i} = T_{\nu f} \equiv T_\nu$$

$$T_\nu = \left(\frac{3}{11}\right)^{1/3} T_\gamma$$

BBN

1947: Alpher, Gamow, "Bethe"

important reactions happening: $\nu p^+ \longleftrightarrow n e^+ \qquad \nu n \longleftrightarrow p^+ e^-$

protons & neutrons really heavy & Boltzmann suppressed after e^+e^- annihilation

$$n_p^{eq} = g_p \left(\frac{m_p T}{2\pi}\right)^{3/2} \exp\left[-\frac{(m_p - \mu_p)}{T}\right]$$

$$n_n^{eq} = g_n \left(\frac{m_n T}{2\pi}\right)^{3/2} \exp\left[-\frac{(m_n - \mu_n)}{T}\right]$$

$T \gg m_e V \int d^3p$

$m_p \approx m_n \qquad Q \equiv m_n - m_p \approx 1.3\,\text{MeV}$

mass difference starts being impt.

correspond w/ quark "mass"

proton: $u^{+2/3}\; u^{+2/3}\; d^{-1/3}$

neutron: $u^{+2/3}\; d^{-1/3}\; d^{-1/3}$

$$Q = (2m_u + m_d) - (2m_d + m_u) = m_u - m_d \approx 1.3\,\text{GeV}$$

$\left(\frac{n_n}{n_p}\right)_{equilibrium} = ?$ $\quad$ @ $T \simeq T_{dec}$

$n_B = (n_p - n_{\bar{p}}) + (n_n - n_{\bar{n}}) = \eta \cdot n_\gamma$

$\mu_p = \mu_n$

$$\left(\frac{n_n}{n_p}\right)_{eq.} = e^{-(m_p - m_n)/T}$$

@ high temp, ratio $\to 1$, doesn't matter until $T < Q$

$$= e^{-Q/T} \longrightarrow \text{mass fraction for neutron} \longrightarrow X_n \equiv \frac{n_n}{n_p + n_n} = \frac{e^{-Q/T}}{1 + e^{-Q/T}}$$

$$X_n(t_{dec}) = \frac{e^{-Q/T_{dec}}}{1 + e^{-Q/T_{dec}}} \approx 1/6$$

Neutrons Decay!

(free neutron)

$n \to p^+ \quad 0.938\ \text{GeV}\ (m_p)$

$\quad\ \to e^- \quad 0.5\ \text{MeV}\ (m_e)$

$\quad\ \to \bar{\nu}_e \quad \sim 0\ \text{MeV}$

$0.939\ \text{GeV}\ (m_n)$

lower mass when putting neutron into nuclei: w/ binding energy 1 MeV

in atomic physics, sum of parts > total $\longrightarrow$ mass defect

allows for higher nuclei! woo

$\tau_n = 890\text{s}$ (neutron lifetime). not enough time for decay yet

ν & γ are just spectators, only p^+, n, e^- trace amounts, set cosmic expansion

$$X_n(t) = X_n(t_{dec}) \cdot e^{-(t - t_{dec})/\tau_n}$$

$\tau_n \gg t_{dec}$

900s $\quad$ 1s

$$X_n(t) \to 1/6 \cdot e^{-t/\tau_n}$$

deuterium $\ ^2H$

now only nuclear reactions $\longrightarrow p + n \leftrightarrow D + \gamma$

2.2 MeV $\approx$ Binding energy

$\mu_p\ p^0 + \mu_n\ n^0 \longrightarrow \mu_D\ D + \gamma\ (\mu_\gamma = 0)$

$D + \gamma \longrightarrow p + n$

prob. of interaction is high

equilibrium still useful approximation

$\mu_p = \mu_n \qquad \mu_D = \mu_p + \mu_n$

now that $m_n + m_p > m_D \rightarrow$ neutrons don't freely decay, stays in nucleus

$$B_D \equiv m_n + m_p - \underbrace{m_D}_{2.4\,\text{MeV}} = 2.2\ \text{MeV}$$

Deuterium fraction?

$$\left(\frac{n_D}{n_p \cdot n_n}\right)_{eq} = \boxed{\frac{g_D}{g_p\, g_n}} \left(\frac{m_D}{m_p \cdot m_n} \cdot \frac{2\pi}{T}\right)^{3/2} e^{B_D/T}$$

$m_D = 2 \cdot m_p \cdot m_n$

$$\rightarrow \left(\frac{n_D}{n_p}\right)_{eq} = \boxed{\frac{3}{4}} (n_n)_{eq} \cdot \left(\frac{4\pi}{T}\right)^{3/2} e^{B_D/T}$$

$(n_n)_{eq} \rightarrow \propto \eta \cdot n_\gamma$

$\eta \approx 10^{-9}$, $n_\gamma \rightarrow T_\gamma^3$

$\Longrightarrow$ n_D not impt until $B_D \gg T$

need $e^{B_D/T} \approx 10^{10}$ for $n_D \simeq n_p$

happens @ $T_{nucl.} \equiv 0.06$ MeV & $t_{nucl} \approx 300\text{s}$ $= \frac{1}{2 \cdot H(T_{nuc})}$

some neutrons decayed before making Deuterium

$$X_n(t_n) = X_n(t_{dec}) \cdot e^{-t_n/\tau_n} \approx \tfrac{1}{6} \cdot e^{-340\text{s}/890\text{s}} \approx \tfrac{1}{6} \cdot \tfrac{6}{8}$$

$$X_n(t_n) \approx 1/8$$

Deuterium as f^n of # neutrons? $\left(\frac{n_D}{n_p}\right)_{eq} = \frac{3}{4} (n_n)_{eq} \left(\frac{4\pi}{m_p T}\right)^{3/2} e^{B_D/T}$

But now other reactions are gonna dominate

$$D + p^+ \longleftrightarrow {}^3He + \gamma$$
$$D + {}^3He \longleftrightarrow {}^4He + p^+$$

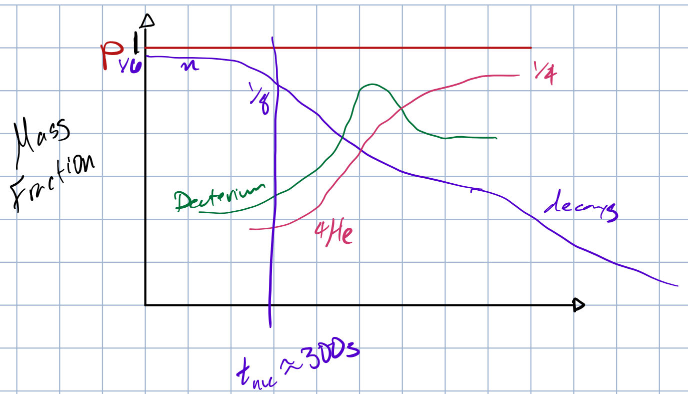
(particularly strong binding energy)

nearly all neutrons $\rightarrow$ ${}^4He$ "Alpha particle"

$$\left(\frac{n_{4He}}{n_p}\right)_{@t_{nuc}} \approx \frac{1}{2} X_n = 1/16$$

each He needs 2 neutrons

$$Y_{4He} = 4\left(\frac{n_{4He}}{n_p}\right) \approx 1/4$$



Mass Fractions

p : 3/4
^4He : 1/4
^3He, D : 10^{-5}
^7Li, ^7Be : 10^{-10}

inputs for BBN?

① initial n/p ratio 1/6 $\Longrightarrow$ $T_{dec.}$, "set by weak force" in lab known in 50s

② reaction rates $\Longrightarrow$ $p+n \leftrightarrow D+\gamma$ $D+{}^3He \leftrightarrow {}^4He+p$ "lab measured rates"

③ "expansion rate" $\Longrightarrow$ set by ν & γ

④ $\eta \equiv \frac{n_B}{n_\gamma} \approx \frac{n_p+n_n}{n_\gamma} \approx 10^{-9}$ $\Longrightarrow$ fit! by spectroscopic astrophysical observation