ν-dec $\longrightarrow$ $e^+e^- \to \gamma\gamma$ $\longrightarrow$ BBN $\longrightarrow$ H-formation $\longrightarrow$ γ-decouple $\longrightarrow$ now

have assumed equilibrium, generally all have same Temp.

Boltzmann Eq: $\quad 1+2 \longleftrightarrow 3+4$

$$\underbrace{\frac{dn_1}{dt} + 3Hn_1}_{\frac{1}{a^3}\frac{d(a^3 n_1)}{dt}} = -\underbrace{\langle\sigma v\rangle}_{\text{particle phys}}\Big[\underbrace{n_1 n_2}_{\text{annihilation}} - \underbrace{\left(\frac{n_1^{eq} n_2^{eq}}{n_3^{eq} n_4^{eq}}\right) n_3 n_4}_{\text{production rate}}\Big]$$

$\left(\frac{n_1^{eq} n_2^{eq}}{n_3^{eq} n_4^{eq}}\right) \longrightarrow$ just some functions

$[\sigma] = A = cm^2 = GeV^{-2}$

$[v] = cm/s$

$[n_i] = cm^{-3}$

Apply to Recombination

$e^- + p^+ \longrightarrow H + \gamma$ $\qquad$ Saha eqn: $X_e = \frac{n_e}{n_p + n_H} \approx \frac{n_e}{n_b} \rightarrow n_e \approx n_b \cdot X_e$

initial condition: $\quad \left(\frac{1-X_e}{X_e^2}\right)_{eq} = \eta \, \frac{2\zeta(3)}{\pi^2}\left(\frac{2\pi T}{m_e}\right)^{3/2} e^{B_H/T}$

$$\frac{1}{a^3}\frac{d(a^3 n_e)}{dt} = \frac{1}{a^3}\frac{d}{dt}(a^3 n_b \cdot X_e) = n_b \frac{d}{dt}(X_e)$$

$\hookrightarrow n_b \propto T^{-3} \propto a^{-3}$

$\eta = \frac{n_b}{n_\gamma}$

$$-\langle\sigma v\rangle \left(n_e n_p - \left(\frac{n_e^{eq} n_p^{eq}}{\cancel{n_\gamma^{eq}} n_H^{eq}}\right) n_H \cancel{n_\gamma} \right)$$

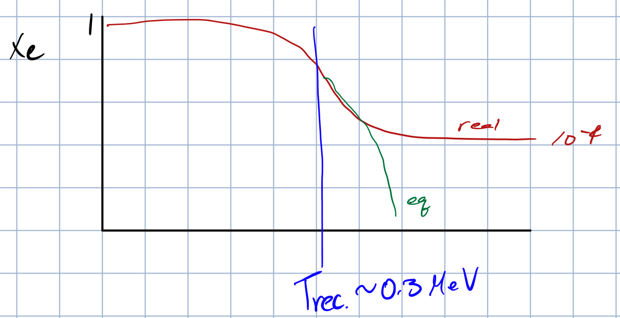
$n_e^{eq} n_p^{eq} \longrightarrow (X_e^{eq})^2 (n_b)^2$

$n_e n_p = (n_b X_e)^2$

$n_\gamma = n_\gamma^{eq}$

$n_H = n_H^{eq}$

$$n_b \cdot \frac{d}{dt}(X_e) = -\langle\sigma v\rangle \, n_b^2 \left(X_e - \underbrace{(X_e^{eq})^2}_{\text{Saha}}\right)$$



now want to go back in time

When was dark matter made?

speculate $T \gg 100$ GeV (technically just need $T > \text{MeV}$ for before video)

$g_*(T) = g_{*s}(T) = \text{const}$

back to radiation domination

$$H^2 = \frac{8\pi}{3} G \rho_{rad} = \frac{8\pi}{3} G \left(\frac{\pi^2}{30} g_* T^4 \right)$$

$$H \approx 1.66 \sqrt{g_*} \frac{T^2}{m_{pl}}$$ $m_{pl} \rightarrow 1.22 \cdot 10^{19}$ GeV

Dark Matter: χ

was χ is eq?

if so, whats the implication? $\rightarrow$ $\chi \bar{\chi} \longleftrightarrow f\bar{f}$

$f\bar{f} \rightarrow$ Standard Model particles $\underbrace{f_p = \{e^-, \mu, t, \dots\}}_{\text{common eq.}}$

how does this rate compete w/ H?

assume perfect symmetry: $\chi = \bar{\chi}$ & $\mu_\chi = 0$

$$n_\chi^{eq} = g_\chi \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp[E/T] + 1}$$

$$= \frac{3}{4} g_\chi \frac{\zeta(3)}{\pi^2} T^3 \qquad (T \gg \text{MeV})$$

$$\longrightarrow g_\chi \left(\frac{m_\chi T}{2\pi} \right)^{3/2} e^{-m_\chi / T}$$

$f\bar{f} \rightarrow \chi\bar{\chi}$

happens if: $\underset{\propto T}{KE} \gg m_\chi$

$T \ll m_\chi$ so reaction is favored to one side

$\chi\bar{\chi} \rightarrow f\bar{f}$

when does $\chi\bar{\chi}$ become diluted compared to H?

Apply Boltzmann eq$^{\text{n}}$ w/ initial condition n_χ^{eq}

LHS: $\frac{1}{a^3} \frac{d(a^3 n_\chi)}{dt}$

$$Y = \frac{n_\chi}{s} \rightarrow n_\chi = Y \cdot s = Y \cdot \frac{2\pi^2}{45} g_{*s} \cdot T^3$$

s ← entropy density

total entropy stays constant in expanding universe

$$S = s \cdot a^3 = \text{const}$$

should find some quantity $\propto s$ for Boltzmann eq$^{\text{n}}$

$$\frac{1}{a^3} \cdot \frac{d}{dt}(\underbrace{a^3 \cdot s}_{\text{const.}} \cdot Y) = \frac{1}{a^3} (\text{const}) \cdot \frac{d}{dt}(Y) = \frac{1}{a^3} \cdot s a^3 \frac{dY}{dt} = s \frac{dY}{dt}$$

make dimensionless: $z \equiv \frac{m_\chi}{T}$ measure of time

$$\frac{d}{dt}(z) = -\frac{m_\chi}{T^2}\cdot\frac{dT}{dt} = -\frac{m_\chi}{T}\cdot\left(\frac{1}{T}\frac{dT}{dt}\right) = -z\,\frac{1}{T}\frac{dT}{dt}$$

bc $g_* = \text{const},\ \longrightarrow\ T = \frac{T_0}{a}\ \longrightarrow\ \frac{dT}{dt} = \frac{-T_0}{a^2}\dot{a} = -\frac{T_0}{a}\left(\frac{\dot a}{a}\right)$
$= -T H$

$$\frac{dT}{dt} = -H\cdot T$$

$$\frac{d}{dt}(z) = -z\cdot\frac{1}{T}\cdot -HT = zH$$

$$dz = zH\cdot dt$$
$$dt = \frac{dz}{zH}$$

$$s\frac{dY}{dt} = s\cdot z\cdot H\cdot\frac{dY}{dz} = LHS$$

$$LHS = \frac{1}{a^3}\frac{d(a^3 n_\chi)}{dt} = s\,z\,H\,\frac{dY}{dz}$$

$T = \frac{m_\chi}{z}$

write $s(T)$ & $H(T)$

in radiation dominated universe : $H(T) = 1.66\sqrt{g_*}\,\frac{T^2}{m_{pl}} = \frac{1.66\sqrt{g_*}}{m_{pl}}\cdot\left(\frac{m_\chi}{z}\right)^2$

$$= \frac{1.66\sqrt{g_*}\,(m_\chi)^2}{m_{pl}}\cdot\frac{1}{z^2} = H(m)\cdot 1/z^2$$

$$s(T) = \frac{2\pi^2}{45}g_* T^3 = \frac{2\pi^2}{45}g_*\cdot\left(\frac{m_\chi}{z}\right)^3 = \left(\frac{2\pi^2 g_*(m_\chi)^3}{45}\right) 1/z^3 = S(m)\ 1/z^3$$

$$s\,z\,H\,\frac{dY}{dz} = \frac{s(m_\chi)}{z^3}\,z\,\frac{H(m_\chi)}{z^2}\,\frac{dY}{dz} = \frac{s(m)\cdot H(m)}{z^4}\,\frac{dY}{dz}$$

RHS : $-\langle\sigma v\rangle\left(n_\chi n_{\bar\chi} - \left(\frac{n_\chi n_{\bar\chi}}{\cancel{n_f n_{\bar f}}}\right)_{eq}\cancel{n_f n_{\bar f}}\right)$

$f\bar f$ are "always" in eq. (during relevant time scale)
always relativistic $T \gg m_f$

$$-\langle\sigma v\rangle\left(n_\chi n_{\bar\chi} - n_\chi^{eq} n_{\bar\chi}^{eq}\right)$$

symmetry: $n_\chi = n_{\bar\chi}$

$$-\langle\sigma v\rangle\left(n_\chi^2 - (n_\chi^{eq})^2\right)$$

$(n_\chi)^{eq} = \int\frac{(d^3p)}{(2\pi)^3}\frac{g_\chi}{e^{E/T}+1} \longrightarrow g_\chi\,\alpha\,T^3$ ($T \gg m$)

$\longrightarrow \alpha\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$ ($T \gg m$)

$$-\langle\sigma v\rangle\cdot s^2\left(Y^2 - Y_{eq}^2\right)$$

$$Y_{eq} = \frac{n_\chi^{eq}}{s}$$

LHS = RHS

$$\frac{H(m)s(m)}{z^4}\frac{dY}{dz} = -\langle\sigma v\rangle\left(\frac{s(m)}{z^3}\right)^2\left(Y^2 - Y_{eq}^2\right)$$

$$\frac{dY}{dz} = \frac{-\langle\sigma v\rangle s(m)}{H(m)z^2}\left(Y^2 - Y_{eq}^2\right)$$

$\equiv \lambda$

$T_{freeze} \approx \frac{m_x}{20}$

$\rightarrow z_f \approx 20$

start in $T \gg m_x$

$R_{ann} = n_x \langle\sigma v\rangle \propto T^3$

$H \propto \frac{T^2}{m_{pl}}$

$$\frac{dY}{dz} = -\frac{\lambda}{z^2}\left(Y^2 - Y_{eq}^2\right)$$

for $z \approx z_{freeze}$ $\quad Y_{eq} \propto e^{-m/T} = e^{-z}$ $\rightarrow$ can neglect near z_{freeze}

$$\frac{dY}{dz} \approx -\frac{\lambda}{z^2} Y^2$$

$$\frac{dY}{Y^2} = -\lambda \frac{dz}{z^2}$$

integrate $\rightarrow$
$$\int_{Y_{eq}}^{Y_{freeze} = Y_{final}} \frac{dY}{Y^2} = -\lambda \int_0^{z_f = 20} \frac{dz}{z^2}$$

$\lambda = \langle\sigma v\rangle \frac{s(m)}{H(m)}$

$$\frac{1}{Y}\Big|_{Y_i}^{Y_f} = -\lambda \frac{1}{z}\Big|_{z_i}^{z_f}$$

$$\frac{1}{Y_f} - \frac{1}{Y_i} = -\lambda\left(\frac{1}{z_f} - \frac{1}{z_i}\right)$$

Weakly interacting massive particle

$$Y_f = Y_f(\lambda)$$

small $\lambda \rightarrow$ departs much earlier

large $\lambda \rightarrow$ departs much later

"correct density" can be set by measurement of λ

"DM is just heavy 4th neutrino" $\rightarrow$ so what's λ?

$m_x \approx 100$ GeV

$$\langle \sigma v \rangle \approx G_F^2 \cdot m_\chi^2$$

$$= (10^{-5}\,GeV^{-2})^2\,(100\,GeV)^2$$

$$= (10^{-10}\,10^{4}\,GeV^{-2})\left(\frac{m_\chi}{100\,GeV}\right)^2$$

$$= 10^{-6}\,GeV^{-2}\left(\frac{m_\chi}{10^{2}\,GeV}\right)^2$$

need $\langle \sigma v \rangle \approx 10^{-26}\,cm^3/s$ or $10^{-9}\,GeV$

assume heavy mass & weak force $\rightarrow$ sets $\langle \sigma v \rangle$ $\rightarrow$ sets λ $\rightarrow$ sets ρ_χ

WIMP Miracle